Dynamic Effects of Muscle Moment Arm Variation and Heavy External Loads on Hinge Joints

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A two-dimensional model has been developed to predict and explain the effects of the variation of muscle moment arms during dynamic exercises involving heavy external loads. The analytical dependence of the muscle moment arm on the joint angle and on the origin and insertion position was derived for an ideal uniaxial hinge joint, modeling the muscle as a cable following an idealized minimum distance path from the origin to insertion that wraps around the bony geometry. Analytical expressions for the ratios of muscular force and the joint restraining reaction components to the external load weight were deduced, for isokinetic and static exercises, as a function of joint angle, joint angular velocity, and the other geometric parameters defining the model. Therefore, external load weight, joint angular velocity, and constraints to joint range of motion may be adjusted reciprocally in order to control in advance the peak value of the components of the joint load during isokinetic exercises. A dynamic formulation of forearm flexion/extension was solved numerically under the condition of constant biceps force in order to highlight the key role played by the variation in muscle moment arm in preventing injury during lifting of external loads against gravity. For example, our analysis indicates that the mean and peak resultant joint loads decrease by 5% and 14%, respectively, as a result of the change in muscle moment arm that occurs over the range of motion.

Key Words: dynamic model, joint loads, elbow

Introduction

The study of the relationships between muscle actions, external loads, ligament tension, and joint contact forces during dynamic exercises represents an important subject matter for therapists and trainers in designing a rehabilitation and strengthening program. In vivo investigations are limited by difficulties in implementing...
accurate measurements of muscle and joint forces. In vitro experiments have provided a wealth of experimental data, however, which can serve as guidelines to real dynamic exercises and activities. In recent years, analytical modeling and numerical simulation have been used extensively to study the mechanics and control of single- and multijoint movement (e.g., Feltner & Dapena, 1989; Nigg & Herzog, 1999). Due to the complexity of the problem, such studies usually refer to free movements or to isometric and quasi-static exercises. Dynamic exercises involve joint forces which, besides muscle forces and external loads, generally depend on the instantaneous kinematic condition of the system; thus their characterization requires explicit solutions of the lagrangian equations.

The present work was aimed at developing an analytical study for a simple and highly idealized two-dimensional dynamic model: an ideal uniaxial hinge joint of cylindrical symmetry activated by the simultaneous action of a single muscular force and a heavy external load fixed to the moving anatomic segment. The muscle moment arm (MA) is evaluated by the origin (O) – insertion (I) method, modeling the muscle as a cable following an idealized minimum distance path from O to I that wraps around the bony geometry. This procedure may produce substantial variations of the muscle MA within the joint range of motion, depending on the O and I position. This prediction agrees with the results of previous theoretical and experimental investigations, such as those which refer to the muscles involved in forearm flexion/extension (Amis, Dowson, & Wright, 1979; An, Hui, Morrey, Linscheid, & Chao, 1981; An, Takahashi, Harrigan, & Chao, 1984; Ettema, Styles, & Kippers, 1998; Murray, Delp, & Buchanan, 1995; Van Zuylen, Van Velzen, & Danier van der Gon, 1988; Winters & Kleweno, 1993). The adopted joint model is applied to biceps-controlled forearm flexion/extension in order to examine the combined dynamic effects of the muscle MA variation and the external load. The specific goal of the present work was to check:

- How muscle MA variation, joint angular velocity, and external load weight each influence joint loads and muscular force intensity in isokinetic exercises;
- How muscle MA variation, external load weight, and muscular force intensity each influence joint kinetics and joint loads in isotonic exercises;
- Whether the variation of the muscle MA around its mean value enhances the lifting efficiency in protecting the elbow joint from injury.

The answers to these questions would provide useful biomechanical information about the type of exercise, the external loads, and the constraints to the joint range of motion. These combined factors allow intensity levels to be consistent with given prescriptions for muscular force and for the components of the joint load. Muscle moment arms are an integral aspect of biomechanical models, and for an accurate interpretation of the results, it is important to understand how different techniques for representing moment arms influence model output.

**Methods**

A geometrical sketch of the 2-D musculoskeletal model of a uniaxial hinge joint in a vertical plane is shown in Figure 1. The joint center of rotation C is assumed to be fixed and is the origin of the Cartesian reference system shown in the figure. Due to joint constraints, a rigid body of mass $M$ composed of a given anatomic segment and a heavy external load is only allowed to rotate around C. The position
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of this rigid system is referred to the angle $\theta$ (joint angle) between $\hat{j}$ and $\hat{u}$, counted counterclockwise from $\hat{j}$ to $\hat{u}$, where $\hat{j}$ is the $y$-axis versor and $\hat{u}$ is the unit vector of a half-line (typically a bony axis) outgoing from $C$ and integral with the rigid system. The mechanical behavior of the system is controlled by the combined action of three different forces:

1. The weight of the system equivalent to the vector $Mg\hat{a}_G$ applied to the system center of mass $G$. We denote by $r_G$ the distance between $G$ and $C$, and by $\varepsilon_G$ the angle of $\hat{u}$ and the vector $CG$, estimated counterclockwise from $\hat{u}$ to $CG$.

2. A muscular force $F\hat{a}_r$ applied at $I$. We denote by $r_I$ the distance between $I$ and $C$, $\varepsilon_I$ is the angle of $\hat{u}$ and the vector $CI$, estimated counterclockwise from $\hat{u}$ to $CI$, and $\varphi$ (traction angle) is the angle of $\hat{a}_r$ and $\hat{u}$, estimated counterclockwise from $\hat{a}_r$ to $\hat{u}$.

3. The joint constraining reaction force $\varphi\hat{a}_\phi$, defined as the resultant of the joint bone-to-bone contact forces and the forces activated by ligament tension. The assumption is that $\varphi\hat{a}_\phi$ is characterized by a negligible moment with respect to the joint rotation axis. Joint forces also impose limits, $\varphi_{\text{inf}}$ and $\varphi_{\text{sup}}$, to the joint range of motion.

The dynamic equation of the system is obtained by projecting the second cardinal equation on the joint rotation axis ($z$-axis) and is given by

$$I_Z \ddot{\theta} = -Mg r_G \sin(\theta + \varepsilon_G) + Fr_I \sin(\varphi + \varepsilon_I)$$

(1)

where $a_G = r_G \sin(\theta + \varepsilon_G)$ and $a_F = r_I \sin(\varphi + \varepsilon_I)$ are the weight and muscle MA, respectively, $I_z$ is the moment of inertia of the system that can be expressed in terms of its gyration radius $\rho$

$$I_z = M\rho^2$$
Joint forces can be calculated by projecting the first cardinal equation \( Ma_G^2 = \vec{F} + Mg + \phi \) along the tangent \((t)\) and principal normal \((n)\) directions relative to the (counterclockwise oriented) circular trajectory of \(G\):

\[
Mr_G \ddot{\theta} = -Mg \sin(\theta + \varepsilon_G) + F \sin(\varphi + \varepsilon_G) + \phi_t
\]

\[
Mr_G \dot{\theta}^2 = -Mg \cos(\theta + \varepsilon_G) + F \cos(\varphi + \varepsilon_G) + \phi_n
\]

According to the sign of \(\phi_n\), \(\phi\) has a tractional \((\phi_n > 0)\) or compressive \((\phi_n < 0)\) character.

To solve Equation 1, the traction angle \(\varphi\) has been expressed in terms of the joint angle \(\theta\) making use of an idealized origin-insertion model of the musculo-skeletal system based on the method developed by Van der Helm (1994). It is assumed that the musculo-tendon actuator can be represented by a cable following the minimum distance path connecting \(O\) and \(I\), both approximated by a single point. Bony geometry, eventually intervening between \(O\) and \(I\), is modeled as a cylinder of radius \(r_j\), whose center coincides with the joint center of rotation \(C\). Thus, the cable follows a straight line between \(O\) and \(I\) if this line is external (Figure 1a) or tangent (Figure 1b) to such a cylinder. On the other hand, if the straight line joining \(O\) and \(I\) passes through the cylinder, then the cable path wraps around its circular contour so as to minimize the total distance between \(O\) and \(I\) (Figure 1c). Under these simplifying assumptions the traction angle \(\varphi\) turns out to be (see Appendix A)

\[
\varphi = \begin{cases} 
\arcsin \frac{r_j}{r_I} - \varepsilon_I & (\theta \leq \theta^*) \\
\theta - \arctan \frac{r_I \sin(\theta + \varepsilon_I) - x_O}{r_I \cos(\theta + \varepsilon_I) + y_O} & (\theta > \theta^*)
\end{cases}
\]

Here \(x_O\) and \(y_O\) are the coordinates of \(O\), which has been assumed to be fixed, and the critical joint angle \(\theta^*\) is given by

\[
\theta^* = \arcsin \frac{r_j}{r_I} - \varepsilon_I + \arcsin \frac{r_j}{r_O} - \arcsin \frac{x_O}{r_O}
\]

where \(\sqrt{r_O} = x_O^2 + y_O^2\) is the CO distance. From Equation 4 the following analytical expression is obtained for the muscle MA

\[
a_F = \begin{cases} 
r_j & (\theta \leq \theta^*) \\
\frac{r_I \sin(\theta + \varepsilon_I) - x_O + \varepsilon_I}{r_I \cos(\theta + \varepsilon_I) + y_O} & (\theta > \theta^*)
\end{cases}
\]

It takes the constant value \(r_j\) below \(\theta^*\) and reaches its maximum (peak MA), equal to the smaller value between \(r_j\) and \(r_O\), at a joint angle, \(\theta_{\text{max}}\), that depends only on \(O\) and \(I\) location.

The model defined by Equations 1–6 can be solved analytically for exercises defined by the condition of constant joint angular velocity (isokinetic exercises)

\[
\dot{\theta} = \text{const.} \equiv \omega
\]
Static exercises can be formally handled as a special case ($\omega = 0$). Equations 1 and 4 yield the muscular force law consistent with Condition 7

$$\frac{F}{Mg} = \begin{cases} \frac{r_G \sin(\theta + \varepsilon_G)}{r_j} & (\theta \leq \theta^*) \\ \frac{r_G}{r_i} \sin \left(\theta - \arctan \frac{r_j \sin(\theta + \varepsilon_j) - x_G + \varepsilon_I}{r_j \cos(\theta + \varepsilon_j) + y_G + \varepsilon_I} \right) & (\theta > \theta^*) \end{cases}$$

Substituting Equation 8 for Equations 2 and 3, and taking into account Equations 4 and 7, we derive the following expressions for the tangent, $\phi_t$, and normal, $\phi_n$, components of the joint reactive force, normalized to the total weight $Mg$

$$\frac{\phi_t}{Mg} = \begin{cases} \sin(\theta + \varepsilon_G) - \frac{r_G}{r_j} \sin(\theta + \varepsilon_G) \sin \left(\arcsin \frac{r_j}{r_i} - \varepsilon_I + \varepsilon_G\right) & (\theta \leq \theta^*) \\ \sin(\theta + \varepsilon_G) - \frac{r_G}{r_i} \sin(\theta + \varepsilon_G) \sin \left(\arcsin \frac{r_j}{r_i} - \varepsilon_I + \varepsilon_G\right) & (\theta > \theta^*) \end{cases}$$

$$\frac{\phi_n}{Mg} = \begin{cases} \frac{r_G \omega^2}{g} + \cos(\theta + \varepsilon_G) - \frac{r_G}{r_j} \sin(\theta + \varepsilon_G) \cos \left(\arcsin \frac{r_j}{r_i} - \varepsilon_I + \varepsilon_G\right) & (\theta \leq \theta^*) \\ \frac{r_G \omega^2}{g} + \cos(\theta + \varepsilon_G) - \frac{r_G}{r_i} \sin(\theta + \varepsilon_G) \cos \left(\arcsin \frac{r_j}{r_i} - \varepsilon_I + \varepsilon_G\right) & (\theta > \theta^*) \end{cases}$$

The component $\phi_t$ is independent of $\omega$ and is almost insensitive to the muscle MA variation whose effects are completely taken into account by the ratio of two sinusoidal functions which differ one from the other only for the constant $\varepsilon_G - \varepsilon_I \sim 0$ phase lag. For $\varepsilon_I = \varepsilon_G$, Equation 9 takes the same form for either $\theta \leq \theta^*$ or $\theta > \theta^*$

$$\frac{\phi_t}{Mg} = \left(1 - \frac{r_G}{r_I}\right) \sin(\theta + \varepsilon_G) \quad (\varepsilon_I = \varepsilon_G)$$

and $\phi_t$ becomes completely insensitive to both muscle MA variation and muscle wrapping. For $\varepsilon_G = \varepsilon_I = 0$, one has $\phi_I < 0$ ($\phi_I > 0$) for $0 < \theta < 180^\circ$ if $r_I < r_G$ ($r_I > r_G$).

Among the different direct dynamic problems, attention has been focused on that defined by a constant muscular force intensity

$$F = \text{const.}$$
so that the variation of the muscular force axial moment is only due to the variation of the muscle MA. The dynamic equation (Eq. 1), taking into account Equation 4, takes the form

\[
\begin{align*}
\dot{\theta} + \frac{g r^2}{\rho^2} \sin (\theta + \epsilon_\omega) &= \frac{F r_i}{M \rho^2} \quad (\theta \leq \theta^*) \\
\dot{\theta} + \frac{g r^2}{\rho^2} \sin (\theta + \epsilon_\omega) &= \frac{F r_i}{M \rho^2} \sin \left[ \theta - \arctan \frac{r_i \sin (\theta + \epsilon_i) - x_o + \epsilon_\omega}{r_i \cos (\theta + \epsilon_i) + y_o + \epsilon_\omega} \right] = 0 \quad (\theta > \theta^*)
\end{align*}
\]  

(12)

This equation has been solved numerically, with the initial condition \( \theta(0) = \dot{\theta}(0) = 0 \), for different values of the constant ratio \( F/M \) that are expressed in terms of the ratio \( \eta \) of the maximum moment of the muscular force to the maximum moment of the total weight

\[ \eta = \frac{r_i F}{r_o Mg} \]

The dependence of \( \phi_i \) on \( \theta \) can be derived analytically by combining Equations 2 and 4 and inserting in the resulting equation

\[
\phi_i = \left\{ \begin{array}{ll}
\frac{r_o}{g} \dot{\theta} + \sin (\theta + \epsilon_\omega) - \frac{F}{M g} \sin \left( \arcsin \frac{r_i}{r_i} - \epsilon_i + \epsilon_\omega \right) & (\theta \leq \theta^*) \\
\frac{r_o}{g} \dot{\theta} + \sin (\theta + \epsilon_\omega) - \frac{F}{M g} \sin \left[ \theta - \arctan \frac{r_i \sin (\theta + \epsilon_i) - x_o + \epsilon_\omega}{r_i \cos (\theta + \epsilon_i) + y_o + \epsilon_\omega} \right] & (\theta > \theta^*)
\end{array} \right.
\]  

(13)

the expression of \( \overline{\dot{\theta}} \) given by Equation 12. From Equations 3 and 4 one gets \( \phi_n \) component

\[
\phi_n = \left\{ \begin{array}{ll}
\frac{r_o}{g} \dot{\theta}^2 + \cos (\theta + \epsilon_\omega) - \frac{F}{M g} \cos \left( \arcsin \frac{r_i}{r_i} - \epsilon_i + \epsilon_\omega \right) & (\theta \leq \theta^*) \\
\frac{r_o}{g} \dot{\theta}^2 + \cos (\theta + \epsilon_\omega) - \frac{F}{M g} \cos \left[ \theta - \arctan \frac{r_i \sin (\theta + \epsilon_i) - x_o + \epsilon_\omega}{r_i \cos (\theta + \epsilon_i) + y_o + \epsilon_\omega} \right] & (\theta > \theta^*)
\end{array} \right.
\]

(14)

which, due to the dependence on \( \dot{\theta} \), can be evaluated only numerically.

The present model will be applied to constant joint angular velocity (\( \dot{\theta} = \text{const.} \)) and to constant muscular force intensity (\( F = \text{const.} \)) exercises relating to biceps-controlled forearm flexion/extension, and use will be made of averaged and rounded out values of the morphological parameters available in the literature.

As is widely accepted, elbow flexion/extension can be schematized as a uniaxial hinge joint with its axis passing through the centers of the capitulum and the trochlear sulcus (London, 1981). Only the biceps I point on the radius is considered in the present work. Moreover, for the sake of simplicity, in numerical applications it is assumed that \( \rho = r_o \). This condition is approximately satisfied in real exercises whereby, typically, a compact external load is held tightly in one’s
hand and the forearm mass (~ 1 Kg) is negligible with respect to the external load mass. Alternatively, the geometrical shape, dimensions, and mass distribution of the external load must be specified and its contribution to the moment of inertia \( I_z \) must be calculated.

The following values of the morphological parameters are assumed:

\[
\begin{align*}
 r_j &= 2 \text{ cm, } r_I = 4 \text{ cm, } \varepsilon_I = 0, \ x_o = 0, \ y_o = 30 \text{ cm, } \rho = r_G = 30 \text{ cm, } \varepsilon_G = 0. 
\end{align*}
\]  

The joint angle \( \theta \) defines the elbow flexion angle with a joint range of motion (Lemay & Crago, 1996) from full extension (\( \theta_{\text{inf}} = 0^\circ \)) to 160° flexion (\( \theta_{\text{sup}} = 160^\circ \)). Under conditions (Eq. 15), Equations 5 and 6 yield \( \theta^* = 33.8^\circ, \ \theta_{\text{max}} = 97.7^\circ, \) and \( a_F (\theta_{\text{max}}) = r_I = 4 \text{ cm}. \) The value of 4 cm for the peak muscle MA has been selected within the wide range of values (3.5 cm \( \leq a_F \leq 5.0 \text{ cm} \)) reported in the literature (see Murray et al., 1995, for review) in order to make the constant value of \( a_F \) in the \( [\theta_{\text{inf}}, \theta^*] \) range equal to the amplitude of the \( a_F \) variation in the \( [\theta^*, \theta_{\text{max}}] \) range \( a_F (\theta_{\text{max}}) - a_F (\theta^*) = a_F (\theta^*) = 2 \text{ cm}. \) To continue, \( r_j \) has been equalized to the value of \( a_F \) taken at \( \theta = 0 \) from Amis et al. (1979) and Pigeon, Yahia, and Felman (1996), which approximately coincides with the value of \( a_F \) given by Murray et al. (1995) for the smallest investigated \( \theta \).

**Results**

An interesting finding we can draw from the analysis of the isokinetic data (Figures 2 and 3) concerns the muscular force \( F \) and the compressive/tractional component \( \phi_n \) of the joint load. Unlike models that assume a constant muscle MA (\( a_F = r_j \)), a muscular force intensity weakly dependent on \( \theta \) is needed to maintain either

![Figure 2 — Dependence of muscular force intensity \( F \) (normalized to total weight \( Mg \)) on joint angle \( \theta \), under conditions (Eq. 15) and for isokinetic and static exercises, as deduced from present model (plain line), assuming a constant muscle MA equal to \( r_j \) (dashed line) and neglecting muscle wrapping (dotted line). A muscular force intensity weakly dependent on \( \theta \) is needed to maintain isokinetic conditions for \( \theta > \theta^* = 33.8^\circ \). Limits of joint range of motion for elbow flexion/extension are assumed to be given by \( \theta_{\text{inf}} = 0^\circ \) and \( \theta_{\text{sup}} = 160^\circ \).](image-url)
isokinetic or static conditions for $\theta > \theta^*$ (Figure 2). In this range the variation of the muscle MA complies with the variation of the weight MA. Below $\theta^*$, due to wrapping effects, the muscle MA is constant and $F$ inherits by the weight MA a sinusoidal dependence on $\theta$. For $\epsilon_G = 0$, $F$ disappears as $\theta$ approaches 0 (Equation 8). As expected, the results clearly indicate that neglecting muscle-cable wrapping ($r_j = 0$) leads to a rough overestimate of the ratio $F/Mg$ for $\theta \sim 0$. Different from $\phi_t$, completely misleading predictions for $\phi_n$ are obtained for $\theta \leq \theta^*$ or $\theta > \theta^*$ neglecting muscle-cable wrapping or assuming a constant muscle MA, respectively (Figure 3). The present model predicts that $\phi_n$ is a monotonic decreasing (increasing) function of $\theta$ below (above) $\theta^*$ and disappears for two values of $\theta$ around 0 and $\theta_{\text{max}}$. With increasing $\omega$, these values tend to approach one another, thus enhancing the tractional character ($\phi_n > 0$) of $\phi_n$. For $\omega \neq 0$ the curve representing the ratio $\phi_n/Mg$ shifts upward of an amount equal to $r_G \omega^2/g$, which is negligible for slow movements ($\omega \ll 1 \text{ rad/s}$) and reaches a value close to unity for fast movements ($\omega \sim 6 \text{ rad/s}$).

The results of the direct dynamic simulation reveal interesting features associated with joint kinetics and joint loads in isotonic exercises. For any given value $F$ of the muscular force, the maximum load weight $M^*g$ that can be lifted up to $\theta_{\text{sup}}$ is approximately given by $0.141 \cdot F$, since it corresponds to a value of $\eta$ given by $\eta^* \equiv r_j/M^* g r_G \approx 0.943$ (Figures 4 and 5). A finite value of $r_j$ allows a rapid increase of $\theta$ immediately above $\theta = 0$, so that the values reached by $\dot{\theta}$ at $\theta^*$ for $\eta e[\eta^*,1]$ are high enough to overcome a nonfavorable relationship between the muscular and the weight moments ($a_F < a_G Mg$) in extended ranges of $\theta$ above $\theta^*$.
Figure 4 — Time dependence of joint angle $\theta$ for a constant muscular force intensity $F$, under conditions (Eq. 15) and for different values of ratio $\eta = r_I F / r_G Mg$ of maximum moment of muscular force to maximum moment of total weight. Value $\eta^* = 0.943$ defines maximum weight $M^*g = r_I F / r_G \eta^*$ that can be lifted up from $\theta_{\text{inf}} = 0^\circ$ to $\theta_{\text{sup}} = 160^\circ$ for any value of $F$. A quasi-static exercise is obtained around $\theta = 120^\circ$ for $\eta = \eta^*$.

Figure 5 — Joint angular velocity $\dot{\theta}$ as a function of joint angle $\theta$ for a constant muscular force intensity $F$, under conditions (Eq. 15) and for different values of $\eta = r_I F / r_G Mg$. For $\eta \approx 1$ the variation of $\dot{\theta}$ can be approximately neglected above $\theta^* = 33.8^\circ$. 
In the close vicinity of the critical limit $\eta^*$ a quasi-static exercise is obtained around $\theta = 120^\circ$. For $\theta > \theta^*$ the variation of the joint angular velocity can be approximately neglected when $F$ and $M_g$ dimensions are such that their maximum moments are nearly the same ($\eta \sim 1$). In the range $[\theta_m, \theta^*]$, $\phi_t$ and $\phi_n$ are slow-variable negative functions of $\theta$ and increase in modulus with increasing $\eta$ (Figure 6). Equations 2 and 3 suggest that the increment of the traction angle $\phi_t$, induced by muscle-cable wrapping, enhances $|\phi_t|$ at the expense of $|\phi_n|$. The effect is mitigated by the concurrent increments of $\theta$ and $\bar{\theta}$ in the same equations. Above $\theta^*$, $\phi_t$ is still negative and reaches a minimum for a value of $\theta \sim \theta_{\text{max}}$ nearly independent of $\eta$, $\phi_n$ is an increasing function of $\theta$ that becomes positive (of tractional type) for a value of $\theta$ that decreases with increasing $\eta$.

The results of the numerical simulation show that the variation of the muscle MA around its mean value increases the lifting efficiency and decreases considerably the mean and peak intensity of joint load. As can be seen in Figure 7, the maximum load weight $M^*g$ that can be lifted up to $\theta_{\text{sup}}$ is an increasing function of $r_j$ which increases by 20% from $r_j \equiv 0$, that is, neglecting muscle-cable wrapping (Figure 7, ▲), to the adopted value of $r_j = 2 \text{ cm}$ (Figure 7, ■). Moreover, for $r_j \equiv 0$, unrealistically long initial residence times around $\theta = 0$ are obtained from the numerical solution of Equation 12, even for values of $\eta$ sensitively greater than $\eta^*$. If a constant muscle MA equal to $r_j$ is assumed, a linear dependence of $M^*g$ on $r_j$ is found (dashed line). The value of $M^*g$ obtained from the present model (Fig-

![Figure 6 — Dependence of tangent $\phi_t$ and normal $\phi_n$ components of joint force (normalized to total load weight $M_g$) on joint angle $\theta$, for a constant muscular force intensity $F$, under conditions (Eq. 15) and for different values of $\eta = r_f/F / r_g/M_g$. A negative (positive) value of $\phi_n$ defines a compressive (tractional type) joint force.](image)
Figure 7 — Maximum weight $M^*g$ (normalized to $F$) that can be lifted up from $\theta_{\text{inf}}$ to $\theta_{\text{sup}}$ by a muscle force of constant intensity $F$ as a function of $r_j$ (for $r_j = 4 \text{ cm}$, $\varepsilon_I = \varepsilon_G = 0$ and $y_O = \rho = r_G = 30 \text{ cm}$), as deduced from Equation 6 (plain line) and assuming a constant muscle MA equal to $r_j$ (dashed line). Arrow compares the value of $M^*g/F$ obtained from present model (Equations 6 and 15) to that obtained assuming a constant muscle MA equal to its mean value $\bar{a}_F$ in the range $[0^\circ, 160^\circ]$ (Equations 16 and 15). Muscle MA variation and muscle-cable wrapping enhance lifting efficiency.

Figure 7, ■) is greater than that obtained assuming a constant muscle MA equal to the mean value of $a_F$ in the range $[\theta_{\text{inf}}, \theta_{\text{sup}}]$ (Figure 7, ●), given by

$$\bar{a}_F = \frac{1}{\theta_{\text{sup}} - \theta_{\text{inf}}} \left[ r_j(\theta^* - \theta_{\text{inf}}) + \int_{\theta^*}^{\theta_{\text{sup}}} r_j \sin \left( \theta + \varepsilon_I - \arctan \frac{r_j \sin(\theta + \varepsilon_I - x_O)}{r_j \cos(\theta + \varepsilon_I + y_O)} \right) \, d\theta \right] = 2.97 \text{ cm}$$

(16)

The relative increment turns out to be about 4% (see arrow in Figure 7). Surprisingly, as shown in Figure 8, these increments in lifting efficiency (20% and 4%) are associated with a decrease in mean value (3% and 5%) and in peak value (8% and 14%) of joint force intensity $\phi$. It is of primary interest to quantify the absolute values of such decreases. Real exercises may involve external load masses ranging from a few kilograms to more than 20 kg. According to Figure 8, for the fixed value $F = 1000 \text{ N}$ of muscular force, which corresponds with a critical value of the total mass $M^*$ of about 14 kg, the different models predict the following values for the $\phi$ peak intensity: 869 $\text{ N}$ (present model), 1011 $\text{ N}$ ($a_F = \bar{a}_F$ model), and 945 $\text{ N}$ ($r_j = 0$ model). Thus the peak decreases of $\phi$ predicted by the present model turn out to be as high as 142 $\text{ N}$ and 76 $\text{ N}$, respectively.
The objective of this work was to develop a 2-D model of a uniaxial hinge joint to predict and explain the effects of the variations of muscle MA during dynamic exercises involving heavy external loads. The importance of muscle MA variations in biomechanical modeling has been widely recognized, and reliable estimations of muscle MA have been provided (Murray et al., 1995) and recently correlated with bone dimensions (Murray, Buchanan, & Delp, 2002). However, the dynamic implications of such variations are still unclear and an understanding of them is of great relevance, especially in presence of free external loads, which in real exercises typically exhibit strong weight MA variations. The relationship between the values of the muscle and the external weight moment arms, within the joint range of motion, plays a decisive role in determining the position and intensity of joint load peaks as well as lifting efficiency. These parameters are important for the planning of optimum rehabilitation and strengthening programs.

There are several limitations associated with the applicability of the present model. The most notable stems from the assumption that the moment of $\Phi$ can be neglected in Equation 1. Unfortunately, a complete dynamic characterization of joint forces usually cannot be provided even with detailed knowledge of the anatomic structure of a selected joint. However, the small value of the mean $\Phi$ MA, as compared to those of $\Phi$ and $Mg$, and the fact that the moments of these two active

![Diagram](image-url)
forces do not simultaneously take vanishing small values (or cancel each other) in finite ranges of $\theta$, even around $\theta = 0$, suggest that the solutions of the dynamic Equation 1 are not seriously affected by this approximation.

A second limitation is related to the assumption that only the effect of a singular muscular force has been considered. Nevertheless, the simultaneous action of other muscular forces and multiple muscle attachment sites can easily be taken into account within the model just specifying properly the corresponding $r_I$, $\varepsilon_I$, $x_O$, and $y_O$ parameters. Different forces are generally characterized by different values of $\theta^*$. Further simplifications occur in the kinematic sketch of the joint model such as its two-dimensionality, cylindrical symmetry, and fixed center of rotation. In particular, for a real uniaxial hinge joint, a slow muscle MA variation, modulated by the specific shape of the articular surface, is expected below $\theta^*$.

The application of the model to constant $F$ exercises requires a thorough consideration. It is widely accepted that the force a muscle exerts in dynamic conditions is related to the muscle architecture and is a function of the actual muscle length, the muscle fiber contraction velocity, and the muscle activation level which is controlled by the central nervous system (An, Chao, & Kaufman, 1991; Kaufman, An, Litchy, & Chao, 1991). Thus, constant $F$ exercises require a subtle relationship among these parameters which must be produced by a complex modulation of the activation level. Nevertheless, such exercises allow an emphasis on the specific effects of the muscle MA variation on the system kinematics and on the joint loads, which is the main goal of the present work. On the other hand, if a constant activation level is maintained, then the length/tension and force/velocity relationships are to be taken into account in the muscular force law. Such an approach, focusing on the physiological aspects of the muscle, will be the subject of a separate research.

Given the above simplifications, some general features associated with the dynamic effects of the muscle MA variation and heavy external loads on uniaxial hinge joints have been established in this study. Within the framework of the present model, Equations 8, 9, and 10 represent the exact solution of the static and the isokinetic problems. These equations provide the analytical expressions for the ratio of muscular force $F$ and joint reaction components ($\phi_t$ and $\phi_n$) to the total weight $Mg$ as a function of joint angle $\theta$, of joint angular velocity $\omega$, and of the other geometrical parameters defining the model.

As expected, with respect to models that assume a constant muscle MA or neglect muscle-cable wrapping, the present model predicts a substantial reduction around $\theta = \theta_{\text{max}}$ and $\theta = 0$, respectively, of both the muscular force and compressive joint force component (Figures 2 and 3). Most significantly, with respect to static exercises, isokinetic exercises reduce (increase) the intensity of the negative compressive (positive tractional) component $\phi_n$ of the joint load below (above) $\theta_{\text{max}} = 97.7^\circ$, without affecting the tangent component $\phi_t$ of the joint load (Equations 9 and 10). For fast movements, the variation of $\phi_n$ induced by $\omega$ may reach values comparable with the weight of the external load. Moreover, the model predicts that the intensity of $\phi_n$ takes its peak values at $\theta^*$ and $\theta_{\text{sup}}$, and these values are roughly six times greater than the external load (Figure 3). Thus external load weight, joint angular velocity, and constraints to joint range of motion may be adjusted reciprocally in order to control in advance the peak value of the tangent as well as compressive and tractional components of joint load during isokinetic exercises.
The numerical solution of the direct dynamic problem shows that with respect to models that assume a constant mean value of muscle MA or neglect muscle-cable wrapping, the present model predicts a greater efficiency (4% and 20%, respectively) in lifting external loads against gravity, associated with a decrease in mean intensity (5% and 3%) and in peak intensity (14% and 8%) of the resultant joint load $\phi$. Such decreases in the peak value of $\phi$ have been quantified for real exercises, and the obtained values are so high that they cannot be disregarded in any realistic biomechanical model.

References


Appendix A

For $\theta > \theta^*$ the traction angle $\varphi$ can be simply expressed in terms of the joint angle $\theta$, as can be deduced from Figure 9, examining the right-angle triangle IOH with hypotenuse IO. In fact, $HI = HO \tan(\theta - \varphi)$, $HI = r_I \sin(\theta + \varepsilon_I) - x_o$ and $HO = r_I \cos(\theta + \varepsilon_I) + y_o$, thus,

$$r_I \sin(\theta + \varepsilon_I) - x_o = [r_I \cos(\theta + \varepsilon_I) + y_o] \tan(\theta - \varphi)$$

and

$$\varphi = \theta - \arctan \frac{r_I \sin(\theta + \varepsilon_I) - x_o}{r_I \cos(\theta + \varepsilon_I) + y_o}$$

For $\theta \leq \theta^*$ (Figure 1) $r_j = r_I \sin(\varphi + \varepsilon_I)$, thus,

$$\varphi = \arcsin \frac{r_j}{r_I} - \varepsilon_I$$

Figure 9 — Geometrical model showing the relationship between $\theta$ and $\varphi$ for $\theta > \theta^*$.

Appendix B: Nomenclature

Points
C Joint center of rotation
G Center of mass of the system
I Muscle insertion point
O Muscle origin point

Distances
$a_F$ Muscle moment arm
$\bar{a}_F$ Mean value of $a_F$ (cont.)
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r_G$</td>
<td>CG distance</td>
</tr>
<tr>
<td>$r_I$</td>
<td>CI distance</td>
</tr>
<tr>
<td>$r_j$</td>
<td>Minimum value of $a_F$, i.e., value of $a_F$ when muscle wrapping occurs</td>
</tr>
<tr>
<td>$r_O$</td>
<td>CO distance</td>
</tr>
<tr>
<td>$x_O, y_O$</td>
<td>Coordinates of O</td>
</tr>
<tr>
<td>$\rho$</td>
<td>Gyration radius of the system</td>
</tr>
</tbody>
</table>

**Angular Quantities**
- $\varepsilon_G$: Angle of $\hat{u}$ ($\hat{u}$ = bony axis versor) and the vector CG
- $\varepsilon_I$: Angle of $\hat{u}$ ($\hat{u}$ = bony axis versor) and the vector CI
- $\varphi$: Traction angle
- $\theta$: Joint angle
- $\theta_{\text{inf}}, \theta_{\text{sup}}$: Limits of the joint range of motion
- $\theta^*$: Maximum value of $\theta$ for which muscle wrapping occurs
- $\theta_{\text{max}}$: Value of $\theta$ for which the muscle moment arm is maximum
- $\dot{\theta}$: Joint angular velocity
- $\ddot{\theta}$: Joint angular acceleration
- $\omega$: Constant value of the joint angular velocity in isokinetic exercises

**Dynamic Quantities**
- $\vec{F}$: Muscular force
- $\vec{Mg}$: Weight of the system
- $\vec{\phi}$: Joint reactive force
- $\phi_n$: Normal component of the joint force
- $\phi_t$: Tangent component of the joint force

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