Biomechanics of Off-Center Monoarticular Exercises With Lever Selectorized Equipment

Andrea Biscarini

We have developed a 2-D analytical biomechanical model for monoarticular open kinetic-chain exercises with lever selectorized equipment, and different relative placement between the joint center of rotation (J) and the center of rotation (C) of the resistance input lever (“off-center” exercises). All the relevant geometrical aspects of such exercises have been characterized: the change with the joint angle of the distance between the resistance pad (P) and J, and of the angle between CP and JP (i.e., the angle between the resistance input lever and the exercising limb). These changes may strongly affect the joint load and the muscle torque in inverse dynamic problems, given the joint kinematics and the mass of the selected weight stack. Therefore, the muscle torque, the shear and axial components of the joint load have been calculated analytically as a function of the relative positioning of C and J, and the length CP, in addition to the parameters that define the joint kinematics, the equipment mechanics, and the external load. From these results we have derived the optimal cam profiles for “off-center” exercises, as well as the geometrical “off-center” setting that minimizes the shear component of the tibiofemoral joint load in leg extension equipment.

Keywords: joint load, ACL, knee, torque, cam

Monoarticular open kinetic-chain exercises with lever selectorized equipment are widely used during the latter stages of rehabilitation programs, as well as during the beginning stages of strength training programs. These devices are easy to learn and use, predefine the user’s movement in a safe and effective form, ensure adequate body stabilization, require minimal supervision, and, by use of cam and pulley systems, provide a variable resistance torque that typically reproduces the average user’s strength curve. Some essential adjustments are built into the mechanical design to give a comfortable fit to individuals of different sizes, and to establish the correct coincident alignment between the joint axis (j-axis) and the rotational axis of the resistance input lever (m-axis). In spite of this, the j-axis is never perfectly aligned to m-axis due to the step-type adjustments, and to the joint anatomy that always involves small and complex displacements of the joint axis within the joint range of motion (ROM). This misalignment induces a change of distance between the joint and the resistance pad during the exercise, that is, a displacement of the resistance pad along the exercising limb. For this reason, the pad usually has a cylindrical shape and can rotate around its longitudinal axis, enabling it to roll on the surface of the limb.

It is commonly known that in leg extension equipment a change in the resistance pad position along the lower leg yields a change in the shear component of the tibiofemoral joint load (Johnson, 1982; Jurist & Otis, 1985; Malone, 1986; Nisell et al., 1989; Zavatsky & O’Connor, 1993, 1995, Zavatsky et al., 1994). Near full extension, a change in pad placement also induces a relevant change in hamstrings activity and consequently in the compressive tibiofemoral force (Li et al., 1999). Very recently, the position of the resistance pad along the shank that minimizes the tibiofemoral joint load has been calculated taking into account all the relevant parameters defining the knee-joint kinematics (joint angle, joint angular velocity and acceleration) and the leg-extension equipment mechanics (cam/pulley system geometry, mass and mass distribution of the mobile equipment elements), as well as the level of external load (mass of the selected weight stack) (Biscarini, 2008). The results highlight that the anterior cruciate ligament (ACL) load can be suppressed, without increasing the posterior cruciate ligament (PCL) load, through a controlled radial displacement of the resistance pad along the lower leg within the ROM.

Monoarticular open kinetic-chain exercises with lever selectorized equipment and j-axis intentionally displaced with respect to m-axis (off-center exercises) offer a simple way of moving the resistance pad along the exercising limb within the ROM. In this work, we have developed an analytical biomechanical model to characterize all the relevant geometrical, kinematical, and dynamical aspects of such off-center exercises. The
specific objective is to examine the dependence on the relative positioning between the j-axis and m-axis of the movement of the pad with respect to the limb; the optimal cam profile, and (for any given values of the relevant biomechanical parameters) of the contact force between the pad and the limb; the joint muscle torque; and the joint reaction force. Finally, the study is aimed at verifying the existence of an off-center setting in leg extension equipment that moves the pad on the lower leg according to the rule recently established (Biscarini, 2008) to minimize the tibiofemoral joint load and avoid ACL stress: for knee flexion angles greater than 40°, the PCL stress is minimized by placing the resistance pad distally along the lower leg; for knee flexion angles smaller than 40°, both the ACL and PCL load can be completely eliminated by continuously moving the resistance pad proximally during knee-extension phase and distally during knee-flexion phase.

Methods

Figure 1 shows an overall geometrical sketch (Figure 1, top) and free body diagrams (Figure 1, bottom) of the user’s limb (L-system); the selected weight stack (W-system); and the mobile mechanical part of the equipment (M-system) constituted by the resistance pad, the cam, and the resistance lever connecting the pad and the cam.

The joint center of rotation (J) is assumed to be fixed at the origin of the adopted Cartesian reference. The external forces acting on the L-system are

- The weight $m_L g$
- The resistance $R$ the pad exerts on the limb
- The muscular forces $F_i$ ($i = 1, 2, \ldots$) of the muscles crossing J
- The joint restraining reaction force $\phi_J$

The rotational dynamic equation of L-system is given by

$$I_L \ddot{\theta} = \sum_i a_F F_i - a_R R - m_L g l_{G_L} \sin \theta$$

where

- $I_L$ is the J-axis moment of inertia of L
- $\theta$ is the joint angle
- $l_{G_L}$ is the distance from J to the center of mass ($G_L$) of L-system
- $a_R$ and $a_F$ are the moment arm of $R$ and $F_i$, respectively

The center of rotation of the M-system is fixed in a point C of coordinates $x_C$ and $y_C$. Due to the different location of J and C, the position of the resistance pad (P) along the exercising limb shifts when the joint angle $\theta$ is changed. The position of P is given by the intersection of its circular trajectory of center C and radius $l_p$

$$(x - x_C)^2 + (y - y_C)^2 = l_p^2$$

and the longitudinal limb axis

$$y = mx, \quad m = \tan(\theta + \pi / 2) = -\cot(\theta)$$

where $m$ is the slope of the limb axis. The following expressions are found for the coordinates of P

$$x_p = x_C + m y_C \pm \sqrt{(x_C + m y_C)^2 - (1 + m^2)(x_C^2 + y_C^2 - l_p^2)}, \quad y_p = mx_p$$

and for the distance $a_R = \sqrt{x_p^2 + y_p^2}$ between P and J

$$a_R = \left| \frac{x_C + m y_C \pm \sqrt{(x_C + m y_C)^2 - (1 + m^2)(x_C^2 + y_C^2 - l_p^2)}}{\sqrt{1 + m^2}} \right|$$
Figure 1 — (a; top) Sketch of the fundamental elements of the lever selectorized equipment and the associated monoarticular open kinetic-chain exercise: (1) user’s exercising limb ($L$-system); (2) selected weight stack ($W$-system); (3) mobile mechanical part of the equipment constituted by the resistance pad, the cam, and the resistance lever connecting the pad and the cam ($M$-system); (4) inextensible cable connecting the weight stack to the cam; (5) massless and frictionless cable redirection pulleys. The joint center of rotation ($J$) does not coincide with the center of rotation ($C$) of the resistance lever. (b; bottom) Free-body diagrams of $L$, $W$, and $M$-systems. The forces acting on these systems are displayed together with the cam moment arm ($a_T$), the moment arm ($a_R$) of $R$, the moment arm ($a_{F_i}$) and the traction angle ($\gamma_i$) of the $i^{th}$ muscular force $F_i$ crossing $J$. 
The $M$-system is subjected to the following external forces:

- The weight $m_M g$
- The force $-R$ the limb exerts on the pad
- The frictionless reactive force $\hat{\varphi}_C$ constraining $M$-system to rotate around $C$
- The force (cable tension) the cable exerts on the cam, equivalent to a force $\hat{T}$ applied at the point where the cable detaches from the cam, and directed as the straight portion of cable connecting the cam and its first redirectional pulley.

The rotational dynamic equation of $M$-system turns out to be

$$I_M \ddot{\theta}_M = Rl_p \cos \beta - a_T T - m_M g l_{G_M} \sin(\theta_M + \varepsilon_M)$$

(6)

where

- $I_M$ is the $C$-axis moment of inertia of $M$
- $\theta_M$ is the equipment rotational angle
- $l_{G_M}$ is the distance from $C$ of the center of mass ($G_M$) of $M$-system
- $\varepsilon_M \approx 0$ is the small angle between $CP$ and $CG_M$
- $a_T$ (cam moment arm) is the $\theta$-dependent moment arm of $\hat{T}$, determined by the geometry of the cam system
- $\beta$ is the angle between the resistance input lever $CP$ and the limb axis $JP$

$$\beta = \arctan \left[ \frac{mx_C - y_C}{\pm \sqrt{(x_C + my_C)^2 - (1 + m^2)(x_C^2 + y_C^2 - l_P^2)}} \right]$$

(7)

The translational dynamics of the $W$-system is described by the equation

$$m_w \ddot{y} = T - m_w g$$

(8)

where $m_w g$ is the weight intensity and $y$ identifies the position along the upward-directed vertical axis of one point arbitrarily selected within $W$-system.

The whole system has one degree of freedom, and the two kinematical variables $\theta_M$ and $y$ in Eqs. 6 and 8 can be expressed as a function of the joint angle $\theta$. As displayed in Figure 1,

$$\theta_M = \theta + \beta$$

(9)

and, consequently,

$$\dot{\theta}_M = \left(1 + \frac{d \beta}{d \theta} \right) \dot{\theta}$$

(10)

$$\ddot{\theta}_M = \frac{d^2 \beta}{d \theta^2} \ddot{\theta}^2 + \left(1 + \frac{d \beta}{d \theta}\right) \ddot{\theta}$$

(11)

$y$ is a monotonic function of $\theta_M$ defined by the geometry of the system constituted by the cam and its first redirectional pulley. When these characteristics are known, $\ddot{y}$ can be given as a function of $\theta_M$ and its time derivatives.

$$\ddot{y} = \frac{d^2 y}{d \theta^2} \ddot{\theta}^2 + \frac{dy}{d \theta} \ddot{\theta}$$

(12)

For example, for a circular cam of radius $r_c$, one has

$$y = r_c \theta_M + \text{const. and } \ddot{y} = r_c \ddot{\theta}_M$$

With use of Eqs. 10 and 11, Eq. 12 becomes

$$\ddot{y} = \left[ \frac{d^2 y}{d \theta^2} \left(1 + \frac{d \beta}{d \theta}\right)^2 + \frac{dy}{d \theta} \frac{d^2 \beta}{d \theta^2} \right] \ddot{\theta}^2 + \frac{dy}{d \theta} \left(1 + \frac{d \beta}{d \theta}\right) \ddot{\theta}$$

(13)
The previous equations determine the unknown forces $T$ and $R$

\[ T = m_w g + m_w \left[ \frac{d^2 y}{d \theta^2} \left( 1 + \frac{d \beta}{d \theta} \right)^2 + \frac{dy}{d \theta} \frac{d^2 \beta}{d \theta^2} \right] \theta^2 + m_w \frac{dy}{d \theta} \left( 1 + \frac{d \beta}{d \theta} \right) \phi \]  \hspace{1cm} (14)

\[ R = \frac{1}{l_p \cos \beta} \left\{ I_M + a_r m_w \frac{dy}{d \theta} \left( 1 + \frac{d \beta}{d \theta} \right) \theta + \right. \]
\[ + \left. \left[ I_M \frac{d^2 \beta}{d \theta^2} + a_r m_w \frac{d^2 y}{d \theta^2} \left( 1 + \frac{d \beta}{d \theta} \right)^2 + a_r m_w \frac{dy}{d \theta} \frac{d^2 \beta}{d \theta^2} \right] \theta^2 + a_r m_w g + m_M g l_{G\phi} \sin(\theta + \beta + \epsilon_M) \right\} \]  \hspace{1cm} (15)

and the total muscular torque \( \sum_i a_i F_i \)

\[ \sum_i a_i F_i = I_L \dot{\theta} + m_L g l_{G\phi} \sin \theta + \frac{a_r}{l_p \cos \beta} \left\{ I_M + a_r m_w \frac{dy}{d \theta} \left( 1 + \frac{d \beta}{d \theta} \right) \theta + \right. \]
\[ + \left. \left[ I_M \frac{d^2 \beta}{d \theta^2} + a_r m_w \frac{d^2 y}{d \theta^2} \left( 1 + \frac{d \beta}{d \theta} \right)^2 + a_r m_w \frac{dy}{d \theta} \frac{d^2 \beta}{d \theta^2} \right] \theta^2 + a_r m_w g + m_M g l_{G\phi} \sin(\theta + \beta + \epsilon_M) \right\} \]  \hspace{1cm} (16)

The singular muscular forces $F_i$ can be determined only by adopting an appropriate optimization procedure (see Nigg & Herzog, 2007, for a review of the optimization methods). Once estimated the optimized values $F_i^*$ of $F_i$, one can get the tangent (shear), $\phi_t$, and normal (axial), $\phi_n$, components of the joint load $\phi_j$ by projecting the first cardinal equation for the $L$-system

\[ m_L \ddot{\theta}_i = \sum_i F_i^* + \dot{R} + m_L g + \phi_j \]

on the tangent ($t$) and principal normal ($n$) of the circular trajectory of $G_i$ (Biscarini, 2003; Biscarini & Cerulli, 2007):

\[ m_L g \ddot{\theta}_i = \sum_i F_i^* \sin \gamma_i - R - m_L g \sin \theta + \phi_i \]  \hspace{1cm} (17)

\[ m_L g \ddot{\theta}_i = \sum_i F_i^* \cos \gamma_i - m_L g \cos \theta + \phi_n \]  \hspace{1cm} (18)

Here, $\gamma$ is the traction angle of $\ddot{F}_i$, defined as the angle of $\ddot{F}_i$ and the longitudinal limb axis ($\gamma = 0$ for a pure axial compression).

When only the main agonist muscle force $\ddot{F}_i$ is taken into account, and the other synergistic and antagonist muscle forces are neglected, the problem becomes determinate, and Eqs. 15 through 18 give

\[ F = \frac{1}{a_F} \left\{ I_L \ddot{\theta} + m_L g l_{G\phi} \sin \theta \right\} + \frac{1}{a_F} \frac{a_r}{l_p \cos \beta} \left\{ I_M + a_r m_w \frac{dy}{d \theta} \left( 1 + \frac{d \beta}{d \theta} \right) \right\} \theta + \]
\[ + \left[ I_M \frac{d^2 \beta}{d \theta^2} + a_r m_w \frac{d^2 y}{d \theta^2} \left( 1 + \frac{d \beta}{d \theta} \right)^2 + a_r m_w \frac{dy}{d \theta} \frac{d^2 \beta}{d \theta^2} \right] \theta^2 + a_r m_w g + m_M g l_{G\phi} \sin(\theta + \beta + \epsilon_M) \right\} \]  \hspace{1cm} (19)

\[ \phi_i = m_L g \ddot{\theta}_i + m_L g \sin \theta - \frac{1}{a_F} \frac{a_r}{l_p \cos \beta} \left\{ I_M + a_r m_w \frac{dy}{d \theta} \left( 1 + \frac{d \beta}{d \theta} \right) \right\} \theta + \]
\[ + \left[ I_M \frac{d^2 \beta}{d \theta^2} + a_r m_w \frac{d^2 y}{d \theta^2} \left( 1 + \frac{d \beta}{d \theta} \right)^2 + a_r m_w \frac{dy}{d \theta} \frac{d^2 \beta}{d \theta^2} \right] \theta^2 + a_r m_w g + m_M g l_{G\phi} \sin(\theta + \beta + \epsilon_M) \right\} \]  \hspace{1cm} (20)
\[
\varphi_n = m_2 l_{G_2} \dot{\theta}^2 + m_1 l_G \cos \theta - \frac{\cos \gamma}{a_F} (l_1 \ddot{\theta} + m_1 l_{G_1} \sin \theta) - \frac{a_R}{a_F} \cos \gamma \frac{1}{l_p \cos \beta} \left( I_M + a_T m_w \frac{dy}{d\theta_M} \right) \left( 1 + \frac{d^2 \beta}{d\theta^2} \right) \dot{\theta} + \\
+ \left[ l_M \frac{d^2 \beta}{d\theta^2} + a_T m_w \frac{d^2 \gamma}{d\theta^2_M} \left( 1 + \frac{d^2 \beta}{d\theta^2} \right) + a_T m_w \frac{dy}{d\theta_M} \right] \dot{\theta}^2 + a_T m_w g + \frac{m_M l_{G_M} \sin (\theta + \beta + \epsilon_M)}{l_p \cos \beta}.
\]

Equations 19 through 21 will be applied to knee extension exercises with leg extension equipment, where \( F \) represents the quadriceps force, and \( \varphi_n \) and \( \varphi_p \) the shear and the axial component of the tibiofemoral joint load, respectively. The dependences on the joint angle of the traction angle \( \gamma \) and the moment arm \( a_R \) of the patellar tendon force were derived from Herzog and Read data (Herzog & Read, 1993). The effect of the weight \( m_M g \) of \( M \)-system was eliminated setting \( l_{G_M} = 0 \) by means of a simple counterbalance to the resistance input arm.

**Results**

Equations 5 and 7 highlight that both the angle \( \beta \) (between \( CP \) and \( JP \)) and \( a_R / l_p \) (of the variable distance between \( J \) and \( P \) to the constant distance between \( C \) and \( P \)) depend only on the three parameters \( x_C / l_p \), \( y_C / l_p \), and \( \theta \). Figures 2 through 7 illustrate the dependence of \( a_R / l_p \) (Figures 2 and 3), \( \beta \) (Figures 4 and 5) and \( \cos \beta / a_R / l_p \) (Figures 6 and 7) on \( \theta \), for different values of \( x_C / l_p \) and \( y_C / l_p \), grouping together curves with the same values of \( x_C / l_p \) (Figures 2, 4, and 6) or \( y_C / l_p \) (Figures 3, 5, 7). All the curves are plotted in the range \( 0 \leq \theta \leq 180^\circ \), and for \( x_C / l_p \) and \( y_C / l_p \) between \(-0.7 \) and \( 0.7 \), so that \( (x_C / l_p)^2 + (y_C / l_p)^2 \leq 1 \); that is, \( U_{CL} \leq I_{CP} \). As such, they represent the upper (+) solution in Eqs. 5 and 7. Once the values of the \( x_C / l_p \) and \( y_C / l_p \) ratios are selected (those that reproduce the desired trend of \( a_R / l_p \) within a given subrange of motion), the value of \( l_p \) determines the range of variation of \( a_R \). When \( (x_C / l_p)^2 + (y_C / l_p)^2 \geq 1 \) (\( U_{CL} \geq I_{CP} \)), \( \beta \) becomes as great as \( 90^\circ \) for a value of the joint angle that depends on \( x_C / l_p \) and \( y_C / l_p \). Here, the resistance input lever \( CP \) becomes perpendicular to the exercising limb \( JP \), and blocks the completion of the exercise. The meaning of the cam correction factor \( \cos \beta / a_R / l_p \) reported in Figures 6 and 7 will be disclosed in the discussion section.

Figure 8 through 10 show the dependence of shear component of the tibiofemoral joint load \( \varphi_2 \) on the joint angle \( \theta \) (and on the standard knee flexion angle \( \theta_{90} = 90^\circ - \theta \)), as deduced from Eq. 20, in the final \( 90^\circ \) of knee extension (\( \theta = 0 \)) (Figure 10), \( \theta = 90^\circ \) (Figure 2), \( \theta = 90^\circ \) (Figure 3) (at full knee extension) in quasi-static conditions, for \( m_g >> m_w \), for an optimized cam profile (see Eq. 22 in Discussion section), and for the following combinations of values of \( x_C \), \( y_C \), and \( l_p \):

\( C \) is below \( J \)

\[
( x_C = 0 \text{ and } y_C = 0, -0.1, -0.15, -0.167 \text{ m} ),
\]

and for each position \( C \) the value of \( l_p \) is such that the pad is placed distally \( (a_R = 0.4 \text{ m}) \) at the beginning of the extension phase \( (\theta = 0) \), when the knee is 90° flexed (Figure 8).

\( C \) is behind \( J \)

\[
( y_C = 0 \text{ and } x_C = 0, -0.1, -0.2, -0.3, -0.4 \text{ m} ),
\]

and for each position \( C \) the value of \( l_p \) such that the pad is placed distally \( (a_R = 0.4 \text{ m}) \) at the beginning of the extension phase \( (\theta = 0) \), when the knee is 90° flexed (Figure 9).

\( C \) coincides with \( J \) and \( l_p = 0.4, 0.3, 0.25, 0.2, 0.165 \text{ m} \) (Figure 10).

In the figure, it is assumed that a positive (negative) shear force \( \varphi_2 \) constrains the tibial plateau posterior (anterior) translation with respect to the femur, thus reflecting a load on the PCL (ACL). A progressive decrease of \( y_C \), \( x_C \), and \( l_p \) in Figures 8, 9, and 10, respectively, produces progressive decrease of ACL stress in the final phase of the extension (up to complete suppression), and concurrent increase of PCL stress. However, the downward displacement of \( C \) gives the smaller PCL stress increase within the whole ROM, thus being more effective in minimizing \( \varphi_2 \).

Figure 11 compares the value of \( a_R \) obtained from Eq. 5 for \( x_C = -0.90 \text{ m} \), \( y_C = -0.74 \text{ m} \), and \( l_p = 1.30 \text{ m} \), with the optimal value, \( (a_R)_{\text{OPT}} \), analytically calculated by Biscarini (Biscarini, 2008), that completely suppresses both ACL and PCL stress \( (\varphi_2 = 0) \) for knee flexion angles smaller than \( 40^\circ \) \( (50 \leq \theta \leq 90^\circ \) ) and \( m_g << m_w \). These values are in excellent agreement with each other, the differences being within \( \pm 4 \) mm in the whole \( 50 \leq \theta \leq 90^\circ \) range. The optimal placement of \( C \) and arm length \( l_p \) were determined, with the guidance of the maps reported in Figures 2 and 3, through a numerical algorithm designed to decrease \( x_C \) and \( y_C \) from the initial values \( x_C = y_C = 0 \), until the desired degree of accuracy for \( a_R = (a_R)_{\text{OPT}} \) is reached. At each step (i.e., for each placement of \( C \)), \( l_p \) is adjusted to the value that gives \( a_R = 0.4 \text{ m} \) (pad placed distally on the lower leg) for \( \theta = 50^\circ \). Among all the different possible solutions, the one corresponding to the minimum value of \( l_p \) was finally selected.
Off-Center Monoarticular Exercises

Figure 2 — Dependence on the joint angle $\theta$ ($0 \leq \theta \leq 180^\circ$) of the ratio $a_R / l_p$ (of the variable distance between $J$ and $P$ to the constant distance between $C$ and $P$) for different values of the parameter $x_c / l_p$. $J$ and $C$ are the centers of rotation of the joint and the resistance lever, respectively. $P$ is the contact point between the exercising limb and the resistance lever pad.

Discussion

New, complex, and expensive devices have been designed in the recent past for rehabilitation applications to control or minimize the overall joint loads, and the stress on specific joint structures, during strengthening exercises. Lever selectorized equipment, for monoarticular open kinetic-chain exercises, may offer a simple and inexpensive way of controlling the joint loads, through a properly selected misalignment between the joint axis and the rotational axis of the resistance input lever. In this work, we have established all the relevant geometrical, kinematical, and dynamical aspects of such off-center exercises.

The different relative placements of $C$ and $J$ produce different sets of displacements of $P$ along the exercising limb within the ROM. According to the selected values of the ratios $x_c / l_p$ and $y_c / l_p$, the distance $a_R$ between $P$ and $J$ may increase or decrease monotonically with $\theta$, and also take a minimum or maximum for any given value of $\theta$ (Figures 2 and 3). Of course, $P$ always describes a circular path around $C$, thus limiting the control of the position of $P$ on the limb with change of $\theta$. Moreover, $l_p$ should always be dimensioned such that $a_R$ does not exceed, within the ROM, the maximum length of the limb useful for the pad support.

Off-center exercises suffer one main drawback: with the exception of the joint angle $\theta$ where $J$, $P$, and...
are aligned, the force \((-\vec{R})\) the limb exerts on the pad is not perpendicular to the resistance input arm \(CP\) (Figure 1, bottom). The component \((R \cos \beta)\) along the perpendicular to \(CP\) produces a rotation of the resistance input arm, whereas the component \((R \sin \beta)\) along \(CP\) statically stresses the equipment, and is compensated by the frictionless reactive force that constrains \(M\)-system to \(C\). For this reason, the values of \(\beta\) are to be selected so as to give values of \(|\beta|\) smaller than 90° within the ROM (Figures 4 and 5). Even in this case, the design of the cam has to be adapted to reproduce the average user’s strength curve. In practice, comparing Relation 16 to the corresponding equation obtained for \(C \equiv J\) (\(\beta = 0\)), one finds that the optimal cam moment arm \(a_r\) in off-center exercises is given by

\[
a_r(\theta) = a_r^*(\theta) \frac{\cos \beta(\theta)}{(a_r(\theta)/l_p)}
\]

(22)

where \(a_r^*\) is the cam moment arm optimized for standard-alignment-exercises \((C \equiv J)\). The cam moment arm is determined by the shape, dimension, and orientation of the cam, and by the location and the radius of its first redirectional pulley (Figure 1, bottom). Plots of optimized \(a_r^*(\theta)\) functions have been recently reported for eight knee extension machines from six different manufacturers (Folland & Morris, 2008), and for a Technogym Selection leg extension equipment (Biscarini, 2008). The values of cam correction factor \(\cos \beta(a_r/l_p)^{-1}\) are immediately deduced from the maps of \(a_r/l_p\) and \(\beta\) reported in Figures 2 through 5. For some combinations of \(x_C/l_p\) and \(y_C/l_p\), the cam correction factor takes values high enough to prevent a practical construction of an effective cam (Figure 6 and 7).

Figures 2 through 7 can be applied to different joints and joint movements, within any subrange of the \(0 \leq \theta \leq 180^\circ\) ROM, thus constituting a powerful tool for the design of off-center exercise devices. As a matter of fact, these maps immediately visualize the values of \(x_C\), \(y_C\), and \(l_p\) that produce a desired change of the contact point between the pad and the limb (Figures 2 and 3);
the corresponding change in the angle $\beta$ between the limb and the resistance input lever, that determines the torque-generating component ($R \cos \beta$) of the force the limb exerts on the lever, and the component ($R \sin \beta$) statically adsorbed by the device (Figures 4 and 5); and, finally, the cam correction functions for the optimization of the muscle force in any given off-center setting (Figures 6 and 7). Moreover, the figures clearly highlight the change of all these curves when $C$ is gradually displaced in horizontal or vertical directions from any off-center staring point.

A controlled change of the contact point between the pad and the limb can be usefully employed to minimize the shear component of the joint load. In leg extension equipment, the ACL and PCL load can be completely suppressed, for knee flexion angles smaller than 40° ($50^\circ \leq \theta \leq 90^\circ$), by continuously moving the resistance pad proximally during knee-extension phase and distally during knee-flexion phase in this joint range (Biscarini, 2008). The optimal distance $(a_R)_{OPT}$ between $J$ and $P$, for appreciably high resistance torques, is nearly independent of resistance level and cam/pulley geometry, and can be approximated by the function

$$\frac{a_{PT}(\theta)}{\sin[\gamma(\theta) + \epsilon_{GI}]}$$

where $a_{PT}$ and $\gamma$ are the moment arm and traction angle of the patellar ligament, $G_{II}$ is the center of the mass of the shank/foot system, and $\epsilon_{GI} \equiv 0$ is the small angle of the longitudinal shank axis and $JG_L$. Surprisingly, this optimal P placement can be reproduced with a high degree of accuracy ($ \pm 4 \text{ mm}$), in the whole $50^\circ \leq \theta \leq 90^\circ$ range, by the following off-center setting: $x_C = -0.90 \text{ m}$, $y_C = -0.74 \text{ m}$, and $l_p = 1.30 \text{ m}$ (Figure 11). The change of the angle $\beta$ between the JP and CP occurring in this condition does not affect the optimal value $(a_R)_{OPT}$ given by (23), supposing that the cam profile has been previously modified according to cam correction factor Eq. 22. For $\theta \leq 50^\circ$ (i.e., $\theta_{\text{flex}} \geq 40^\circ$), this combination of the parameters $x_C$, $y_C$, and $l_p$ yields a value of $a_R$ greater than 0.4 m, which is a rounded mean value of the length of the lower leg. In this range of $\theta$, the pad has...
Biscarini to be placed distally along the lower leg to minimize the PCL load (Biscarini, 2008). Therefore, it is sufficient to prevent the pad from going beyond the foot to realize an off-center exercise that effectively minimizes the tibiofemoral joint load (PCL stress) for knee flexion angles greater than 40°.

The effects of hamstring co-contractions and knee joint accelerations on the cruciate ligaments’ load, and on the optimal value \( \beta_{\text{OPT}} \), have been extensively discussed by the author in a previous paper (Biscarini, 2008). It was clearly demonstrated that the optimal value (23) represents the best compromise for a preventive joint protection, even in the presence of hamstring co-contractions and knee joint accelerations not predictable in advance.

The major limitation of this study stems from the assumption that the joint axis is fixed. A displacement of joint axis within the ROM induces a change in the position of the pad along the limb in addition to the change due to the misalignment between C and J. However, when the CJ distance is great enough to produce a considerable change of \( a_r \), the displacement of J generally induces only a negligible perturbation to this change. Ultimately, this effect can only be taken into account on a precise case-by-case basis, referring to the specific joint and joint movement. In practice, the present model can be applied, at least in the quasi-static regimen, even when the CJ distance is \( \theta \)-dependent, or when the misalignment between J and C is entirely due to the joint axis motion. In real situations, the displacement of J might also depend on the level of external load as well as on the combination of intersubject variability factors. The patellar tendon angle and moment arm also change between subjects (see Tsaopoulos et al., 2006, for a review) and are affected by joint loading (Pandy & Shelburne, 1997) and measurement method (Tsaopoulos et al., 2006) as well. However, the present analytical model enables a straightforward use of any different set of anthropometric data.

In conclusion, a misalignment between J and C induces a complex change within the ROM of the distance between J and P, and of the angle between CP and JP (Eqs. 5 and 7), which may significantly influence the muscular torque and the joint load, as highlighted by Eqs. 16-21. Thus, off-center exercises should be used only for

Figure 5 — Dependence on the joint angle \( 0 \leq \theta \leq 180^\circ \) of the angle \( \beta \) between CP and JP, for different values of the parameter \( y_C / l_p \). \( \beta \) coincides with the angle between the exercising limb and the resistance lever, as their thicknesses have been neglected.
specific needs, once all the relevant biomechanical effects are preventively evaluated. In leg extension equipment, the shear component of the tibiofemoral joint load can be effectively minimized, and ACL stress suppressed, by a specific positioning of C with respect to J. However, even in such cases, a careful adaptation of the cam profile should be taken into account, according to the indications given by Eq. 22.

Figure 6 — Dependence on the joint angle $\theta \ (0 \leq \theta \leq 180^\circ)$ of the cam correction factor $\cos \beta (aR / lP)^{-1}$, for different values of the parameters $x_C / l_P$. The optimal cam profile for off-center exercises can be obtained from the cam correction factor and the optimal cam profile for standard-alignment ($C = J$) exercises (see Eq. 22).

References

Figure 7 — Dependence on the joint angle $\theta$ ($0 \leq \theta \leq 180^\circ$) of the cam correction factor $\cos \beta (\alpha / l_p)^{-1}$, for different values of the parameter $y_c / l_p$. The optimal cam profile for off-center exercises can be obtained from the cam correction factor and the optimal cam profile for standard-alignment ($C = J$) exercises (see Eq. 22).

Figure 8 — Dependence of shear component of the tibiofemoral joint load $\phi_t$ on the joint angle $\theta$ (and on the knee flexion angle $\theta_{\text{flex}} = 90^\circ - \theta$), in the final $90^\circ$ of knee extension ($\theta = 0$ for $90^\circ$ knee flexion, $\theta = 90^\circ$ at full knee extension) in quasi-static conditions, for $m_T >> m_L$, and for values of $x_C = 0$, and $y_C = 0, -0.1, -0.15, -0.1675$ m. For each position of C, the value of $l_P$ is such that the pad is placed distally ($a_R = 0.4$ m) at the beginning of the extension phase ($\theta = 0$), when the knee is $90^\circ$ flexed. Positive (negative) shear forces $\phi_t$ correspond to loads on the posterior (anterior) cruciate ligament.

Figure 9 — Dependence of shear component of the tibiofemoral joint load $\phi_t$ on the joint angle $\theta$ (and on the knee flexion angle $\theta_{\text{flex}} = 90^\circ - \theta$), in the final $90^\circ$ of knee extension ($\theta = 0$ for $90^\circ$ knee flexion, $\theta = 90^\circ$ at full knee extension) in quasi-static conditions, for $m_T >> m_L$, and for values of $x_C = 0$, and $y_C = 0, -0.1, -0.2, -0.3, -0.4$ m. For each position of C, the value of $l_P$ is such that the pad is placed distally ($a_R = 0.4$ m) at the beginning of the extension phase ($\theta = 0$), when the knee is $90^\circ$ flexed. Positive (negative) shear forces $\phi_t$ correspond to loads on the posterior (anterior) cruciate ligament.
Figure 10 — Dependence of shear component of the tibiofemoral joint load $\phi_i$ on the joint angle $\theta$ (and on the knee flexion angle $\theta_{\text{flex}} = 90^\circ - \theta$), in the final $90^\circ$ of knee extension ($\theta = 0$ for $90^\circ$ knee flexion, $\theta = 90^\circ$ at full knee extension) in quasi-static conditions, for $m_p >> m_L$, and for following combinations of values of $x_C = 0$, $y_C = 0$, and $l_p = 0.4, 0.3, 0.25, 0.2, 0.165$ m. Positive (negative) shear forces $\phi_i$ correspond to loads on the posterior (anterior) cruciate ligament.

Positive (negative) shear forces $\phi_i$ correspond to loads on the posterior (anterior) cruciate ligament.

Figure 11 — Comparison between the value of $a_R$ obtained for $x_C = -0.9$ m, $y_C = -0.74$ m, and $l_p = 1.30$ m (plain line), and the optimal value $(a_R)_{OPT}$ (Biscarini, 2008) that completely suppresses both ACL and PCL stress ($\phi_i = 0$ ) in the range $50 \leq \theta \leq 90^\circ$ (knee flexion angles smaller than $40^\circ$) for $m_L << m_p$ (dotted line). The difference between $a_R$ and $(a_R)_{OPT}$, corresponding to these values of $x_C$, $y_C$, and $l_p$, is displayed in the lower part of the figure. The knee flexion angle $\theta_{\text{flex}}$ is given by $\theta_{\text{flex}} = 90^\circ - \theta$. 