Differential Specific Mechanical Energy as a Quality Parameter in Racing Alpine Skiing

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An important question in alpine skiing is how to determine characteristics of well-performed ski turns, an issue that has become more crucial with the arrival of new carving skis. This article introduces a new method for estimating the quality of skiing at each point of observation based on mechanical energy behavior that can be measured using established motion analysis techniques. It can be used for single-or multiple-skier analyses for evaluation of skiing technique as well as racing tactics. An illustration of its use is shown by analyzing 16 top-level racers using a 3-D kinematical system and video recorded during an alpine ski world cup race. Based on energy behavior of several racers, it is demonstrated that the most direct line with shortest radius of turn is not necessarily the most effective strategy in contrast to what some coaches believe.

Keywords: mechanics, alpine skiing, giant slalom, 3-D kinematical measurements, trajectory

The new geometry of skis, namely, carving skis (e.g., Glenne et al., 1997; Nordt et al., 1999; Howe, 2001 etc.), has had a great impact on the turning mechanism (Kugovnik et al., 2003) as well as on racing technique (e.g., Raschner et al., 2001; Supej et al., 2001, 2002; Mueller et al., 2004). World Cup racers are executing carving and pivoting (side skidding) turns. They use single as well as double motion techniques, in which the skier performs a single extension and flexion and double extension and flexion body movement in one turn, respectively (Supej et al., 2002, 2004).

The differences among top-level racers can be very small, for example 0.01 s. On the other hand, it has been proven that intercourse differences or section differences can be quite big, for instance up to 10% (Supej et al., 2004, Supej & Cernigoj 2006). The large differences are especially interesting and mostly arise from different approaches to and tactical abilities of using a carving and pivoting technique (Supej & Cernigoj 2006). Nevertheless, it is still unclear even for professional coaches why exactly and where exactly skiers are losing or winning races. Mostly the skiers’ “mistakes,” such as bad timing or a low trajectory can be identified, but it makes it much harder to identify where exactly the performance was deficient. It was proven that the amplitude of ground reaction forces, the turn radii, velocities, and acceleration are not satisfactory parameters in themselves for estimating the quality of skiing even though all these parameters are essential for racing alpine skiing (Supej et al., 2005a). A skier’s high velocity at a point of observation does not necessarily mean that the skier’s performance is good because he could be slowing down or skiing an inappropriate trajectory. Similarly, a high level of acceleration does not necessarily mean an excellent performance because the skier’s instant velocity could be low and he could be able to accelerate more because of a lower air drag (Supej et al., 2005a). As a consequence, even the use of very sophisticated equipment measuring all these parameters is not enough to solve the puzzle. Therefore, a major issue today is how to determine and measure the quality of a racing ski turn and, relying on these measurements, to be able to decide which technique is better in given conditions. To achieve that, it is necessary to introduce an integrated parameter that would make that possible via mechanical principles.

Our research proposes the use of an energy principle to estimate the quality of a ski turn at each...
point of observation. To demonstrate the efficiency of the proposed criteria and to show one of the potential measuring methods, 3-D kinematical measurements of top-level racers from the giant slalom at the 2003 Pokal Vitranc (the Vitranc Cup) races in Kranjska Gora, Slovenia, are used.

Methods

The method we are introducing here for estimating the quality of a racing ski turn is based on mechanical energy. From the physical point of view, it can be justified that the main motor in alpine racing skiing is potential energy $E_p$. The contribution of the skier’s active movement (extending, push-off) to the overall kinetic energy $E_k$ (velocity) is very small or even negligible (Supej et al., 2001; Kugovnik et al., 2003). This is because of the nature of movement and circumstances where the push off is executed despite the fact that some coaches and experts have different opinions (Petrovič et al., 1987). Even scientists have proposed mechanical models showing this, but they are limited to very low velocities and special conditions, such as Lind and Sanders (1996) with their work on “pumping to increase velocity.” Consequently, when a skier descends a slope, the difference in potential energy $\Delta E_p$ equals the sum of the difference in kinetic energy $\Delta E_k$ and the work of energy dissipation $W_d$:

$$\Delta E_p = \Delta E_k + W_d$$  \hspace{1cm} (1)

The work of energy dissipation in alpine skiing consists of two parts:

$$W_d = \int (F_t + F_f) \, ds$$  \hspace{1cm} (2)

where $F_t$ describes the snow friction and $F_f$ the air drag. In this case, the snow friction encompasses the tribological properties of skis and, most importantly, the (im)precise guiding of skis. The overall mechanical energy $E_{\text{mech}}$ consists of kinetic $E_k$ and potential $E_p$ energy:

$$E_{\text{mech}} = E_k + E_p$$  \hspace{1cm} (3)

They can be derived from the center of gravity’s trajectory $r$ in the simplest approximation:

$$E_k = m(\dot{r})^2/2$$  \hspace{1cm} (4)

and

$$E_p = mg\partial$$  \hspace{1cm} (5)

where $m$ denotes the mass of the skier, $(\dot{r})$ the time derivative of the center of gravity’s trajectory, and $\partial$ the displacement along gravity $g$ of the skier’s center of gravity. Specific mechanical energy $e_{\text{mech}}$ should be calculated for an exact quantitative comparison between racers:

$$e_{\text{mech}} = E_{\text{mech}}/m = (\dot{r})^2/2 + g\partial$$  \hspace{1cm} (6)

If $e_{\text{mech}}$ is calculated at each point of measurement along the run, then the complete energy behavior is known.

The rate of energy dissipation can be calculated by the final parameter differential specific mechanical energy $\text{diff}(e_{\text{mech}})$:

$$\text{diff}(e_{\text{mech}}) = \partial e_{\text{mech}}/\partial \partial$$  \hspace{1cm} (7)

It should be pointed out that this is the altitude derivative $\partial\partial$ and not the time derivative. This approach eliminates the factor of a skier’s exact location on the terrain and his or her velocity.

Further, altitude derivation precisely reflects the use of $E_p$. Simplifying Equations 6 and 7 gives:

$$\text{diff}(e_{\text{mech}}) = \partial(\dot{r})^2/2)/\partial\partial + g$$  \hspace{1cm} (8)

Because we are dealing with discrete data in measurements, the derivatives are substituted by differentials. This is why the name differential was chosen instead of derivative $e_{\text{mech}}$. Since the altitude is (normally) continuously decreasing in alpine skiing, it makes much more sense to change the sign of the $\partial\partial$ in Equations 7 and 8 to have smaller values for inefficient skiing and higher values for efficient skiing. The equation transforms into:

$$\text{diff}(e_{\text{mech}}) = \Delta e_{\text{mech}}/(-\Delta\partial) = -\Delta(v^2/2)/\Delta\partial - g$$  \hspace{1cm} (9)

where $v$ represents the absolute velocity of the skier’s center of gravity.

To illustrate the new principle, an example will be shown on the basis of 3-D kinematical measurements taken at the giant slalom world cup races held in Kranjska Gora (Pokal Vitranc 2003). They were recorded with six internally synchronized Sony DV-CAM DSR-300 PK professional camcorders covering three kinematical subspaces. A pair of camcorders covered one of the subspaces. The camcorders were fixed, lifted higher for better visibility, and positioned between 40 and 70 m away from the course set up to minimize error involving lenses. The size of the pixel from all the camcorders was always larger than 1 and not greater than 2 cm. The overlapping between the subspaces was between two and four recorded frames, depending on the overlapping and each measurement. The speed of the camcorders was 50 Hz, but we downgraded the frequency to 25 frames per second. The resolution was a standard 720 × 576 pixels. In addition, two pan, tilt, zoom Sony mini-DV DCR-TRV 30 E camcorders were used to record the inspected region from both sides. These two recordings were only used to help in gathering kinematical data and for the experts’ video analysis. All subspaces were calibrated using the electronic tachymeter Leica TCR1102 X-Range and eight 1.95-m-long aluminum calibrating poles with special reflecting marks. The calibrating poles were planted along the measuring part of the course up before the race started and were removed before the race. One giant slalom turn with the end of the previous one and the beginning of the next one was measured for all the racers. Two additional poles were planted in the direction of the steepest inclination and in the middle of the inspected area to define the main direction of the fall line. The actual fall
The slope inclination ranged from 24° to 26°. The snow was hard and icy, the visibility was perfect, and the air temperature was around 0°C. Sixteen top-level world cup racers were measured in the first run. The 15-segment model of the skier was digitized and defined by 17 reference points. The segments of the model represented parts of the body, linked with pointlike joints. The masses and centers of gravity of the segments as well as the center of gravity (CG) of the body were calculated as lined segments using body segment parameters from Dempster (1955). The APAS Ariel 3-D kinematical software (Ariel Dynamics Inc., San Diego, CA) was used to transform the 2 × 2D data into 3-D data. The raw data from each subspace were taken from APAS software. The subspaces were joined together with specially designed routines in Matlab. They use the center of mass trajectory and its first differential to check the consistency between the data of different subspaces in the region of the overlapping and its surrounding points. The routines automatically decide where the best stitching point is in the overlapping area and, if necessary, the lower subspace may be shifted adequately. To calculate the presented parameters from the stitched subspaces, we used custom software, namely, the KinSki 3.1 system (Supej et al., 2005b). The point of the beginning and the end of the turn was calculated as a cross-section of the skis’ and CG’s trajectory (Supej et al., 2003). The energy parameters were determined as previously described, and the turning radius of the outside ski was determined with fitting arcs (Supej et al., 2005b). The radius of the arc was calculated at each point of observation from the point itself and the two surrounding points. The section times were calculated from the center of mass positions on the fall line using a spline interpolation. The start trigger was at 0 m and the stop trigger at 45 m on the fall line. There the skiers were descending toward the fall line to minimize trajectory differences. To estimate the error of the measured data, we used the inside poles of the gates. They were first measured with the Leica electronic tachymeter and compared with the 3-D kinematical measurements. The standard error of the location was below 2 cm. Some joint positions could be less accurate owing to common problems in 3-D kinematical measurements based on camcorders, but the accuracy satisfies the requirements for reliable results that show higher differences.

Sixteen top-level world cup racers were included in this study for calculating the mean values and standard deviations for trajectories, specific mechanical energy, and differential specific mechanical energy. The results for four of them, which are more interesting, are stressed and discussed in depth to show the application of the new principle.

**Results**

The new method based on mechanical energy enables a quantitative observation of skiing quality at each point of observation along a run. A skier’s mechanical energy behavior can be roughly described by the diagram in Figure 1. When there is no energy dissipation, \( W_d = 0 \), potential energy is transformed directly into kinetic energy and the graph is a linear horizontal line (Figure 1). In this case, the mechanical energy’s altitude derivative equals zero. When the work of air drag and snow friction is greater than zero, \( W_d > 0 \), energy dissipation is present. In this case, the mechanical energy decreases. Efficiency in the use of potential energy reflects the skier’s technique or ability to minimize snow friction and air drag. When the skis run smoothly, such as in carved turns, the dissipation is smaller. On the contrary, higher energy dissipation characterizes the imperfect running of skis, such as with side skidding.

![Figure 1 — Mechanical energy behavior with (solid line) and without energy dissipation (dotted line)](image-url)
The diagrams in Figures 2 and 3 show one complete turn plus the end of the preceding turn and the beginning of the consecutive turn to be able to inspect more precisely the initial and final states. Two racers with better energy behavior (i.e., better skiing quality) and two racers with higher energy dissipation (i.e., lower skiing quality) are emphasized in Figures 2 and 3, respectively. The whole region of the calculated mean $e_{\text{mech}}$ behavior ranges from 155.41 J/kg to −42 J/kg and has a decreasing tendency as expected (Figures 2a and 3a). The highest $e_{\text{mech}}$ (167.7 J/kg) belongs to F.C. at the beginning of the inspected interval and the lowest to subject J.K. (−50.62 J/kg) at the end of the interval among the emphasized four racers. The $e_{\text{mech}}$ standard deviation ranges between 6 and 11 J/kg. If

Figure 2 — Specific mechanical energy (a), differential specific energy (b) and skiers’ arithmetic mean of ski trajectories in the inspected interval (c) for two chosen skiers who had better energy behavior in the inspected turn—higher skiing quality. Gray shadings with a thin broken line show the mean values and the standard deviation for each parameter. All three diagrams are drawn against the fall line. The vertical dotted lines show the interval where the racers started and ended the turn (weight transfer), whereas the vertical solid line shows the position of the gate (1st run, Kranjska Gora 2003). Note. mean = statistical mean value; C.M. = Christian Mayer; U.P., Uros Pavlovic; Std = standard deviation.
we ascribe the standard deviation only to differences in kinetic energy and calculate the velocity, the range of the standard deviation would be from 3.46 m/s to 4.69 m/s.

The behavior of the $\text{diff}(e_{\text{mech}})$ is not decreasing as the $e_{\text{mech}}$, but it has typical cyclic behavior with maximums around the weight transition, reflecting good energy behavior and minimums around the gate, reflecting higher energy dissipation (Figures 2b and 3b). The mean values range from $-31.55 \, \text{J/kg} \cdot \text{m}$ at the end of the preceding turn to $6.63 \, \text{J/kg} \cdot \text{m}$ during the weight transition and even up to $12.54 \, \text{J/kg} \cdot \text{m}$ at the beginning of the consecutive turn that starts where the terrain changes quickly from flat to step. In this case, the standard deviation ranges from $1.62 \, \text{J/kg} \cdot \text{m}$ to $4.04 \, \text{J/kg} \cdot \text{m}$. The lower values or, in other words,

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**Figure 3** — Specific mechanical energy (a), differential specific energy (b) and skiers’ arithmetic mean of ski trajectories in the inspected interval (c) for two chosen skiers who had a higher energy dissipation in the inspected turn—lower skiing quality. Gray shadings with a thin broken line show the mean values and the standard deviation for each parameter. All three diagrams are drawn against the fall line. The vertical dotted lines show the interval where the racers started and ended the turn (weight transfer), whereas the vertical solid line shows the position of the gate (1st run, Kranjska Gora 2003). Note. mean = statistical mean value; F.C. = Frederic Covili; J.K., Jernej Koblar; Std = standard deviation.
the smallest differences in performance among top-level racers, correspond to the areas of weight transition of approximately 9 m to 13.5 m and 34 to 36.5 on the fall line (Table 1) and around the apex of the turn at around 22 m on the fall line. In all other regions, the standard deviation and therefore the differences in performance are higher.

Recently, one of the most common discussions among coaches is about the line taken by skiers. To evaluate skiers’ lines, the trajectories of the skis are presented in Figures 2c and 3c. The standard deviations were relatively large, ranging from 9 to 70 cm. Further, the trajectory of subject U.P. was 1.07 m away from the mean trajectory at the beginning of the turn at 10 m on the fall line and 1.19 m away at the end of the turn about 35 m down the fall line. The differences in trajectories were even more evident when examining the turn radii in Figure 4. Two racers with better energy behavior, namely U.P. and C.M., have completely different radius characteristics and started much earlier with high-intensity turning, that is, radii below 16 m, than the other two racers with high energy dissipation. Subject U.P. started to turn intensively more than 5 m higher on the fall line than did subjects F.C. and J.K.

**Discussion**

A direct consequence of energetic analysis is that a skier should maintain their high kinetic energy to achieve good results. There is only a finite amount of potential energy available. When a skier has lost their kinetic energy, it is sometimes impossible to increase it again to the same level because they have already used the available stock of potential energy when skiing down the slope. It can be stated that for different disciplines and given conditions friction or drag energy dissipation can play the primary role (e.g., Watanabe & Ohtsuki, 1977; Luethi & Denoth, 1987; Savolainen, 1989; Kaps et al., 1996).

The normal tendency is to lose energy even though racers want to minimize the dissipation as seen in Figures 1, 2a, and 3a. The energy losses are higher around the gate as a result of the stronger ground reaction forces (e.g., Babel et al., 1997; Schaff et al., 1997; Raschner

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**Figure 4** — Turn radii of the skiers’ outside skis in the interval between the beginning and end of the turn*. Racers with good energy behavior, U. Pavlovic and C. Meyer, are marked with a dotted line and a solid line, respectively, whereas racers with a high energy dissipation, F. Covilli and J. Koblar, are marked with circles and stars, respectively (1st run, Kranjska Gora 2003). *Note. C.M. = Christian Mayer; U.P. = Uros Pavlovic; F.C. = Frederic Covili; J.K. = Jernej Koblar. Note. The region of the diagram is shortened to show more precisely that part of the turn with a high rate of turning.
et al., 2001; Supej et al., 2002), the higher rate of turning (Figure 4), and consequently the skis are not running so optimally (e.g., Kaps et al., 1996; Supej et al., 2002). On the other hand, during the weight transition, the function is just the opposite. Skiers are not turning so intensively, the ground reaction forces are weaker, and the skis are running more smoothly. When analyzing the measurements, several curves of the specific mechanical energy from different top-level racers are more or less parallel, yet important differences can be observed. Even when the curves are parallel, the skis are not equally fast. For example, subject J.K.’s $e_{\text{mec}}$ curve is more or less parallel to the mean curve (Figure 2b), but mostly it is on the bottom edge of the standard deviation interval. The only two regions deviating from “parallel behavior” are between 4 and 9 m and 9 to 17 on the fall line. In the first one, he dissipated lots of energy, where diff($e_{\text{mec}}$) was far below the standard deviation interval, and in the second interval diff($e_{\text{mec}}$) it was well over the standard deviation interval, thereby showing the regaining of some kinetic energy. Nevertheless, his section time was approximately 9% longer than the best section time of subject C.M. (Table 1). His skiing “performance” was mostly close to average, but really low in just the one mentioned region, which can be very clearly identified by the new principle. His main reason for a poor time was that he came into this section with low $e_{\text{mec}}$, which he was unable to increase.

The standard deviation values ranging from 6 to 11 J/kg are relatively high, despite the fact that it does not seem so in Figures 2a and 3a. The important thing is that the diff($e_{\text{mec}}$) is very sensitive to the changes that are happening and shows the exact locations where the high energy and low energy dissipations are present. Nevertheless, key differences between the top-level racers are even higher than the size of the standard deviation. Following the curve of subject F.C.’s $e_{\text{mec}}$ in Figure 3a, it can be seen that he entered the inspected area with a large amount of energy and ended up with a very low $e_{\text{mec}}$. The diagram in Figure 3a clearly shows he started to lose it right after the first weight transfer (beginning of the turn) all the way through to the gate, where he started slowly to regain some energy, although he started losing it again at the entrance to the following turn. Most of the time he was dissipating a lot of energy compared with the other racers, being on the bottom of the standard deviation interval or even lower (Figure 3b). In general, his only good parts were around the weight transfer. Even though he finished the section with a satisfactory time (Table 1), he lost most of his energy advantage he held at the start of the inspected section. So the time in this case is obviously not a good parameter to define the quality of skiing.

The explanation can involve looking at his skis’ trajectory (Figure 3c) and his turn radii in Figure 4. He had an average entrance to the inspected turn but performed a very sharp turn with a very direct line to the next gate, starting to turn intensively (radii below 16 m) at 17 m on the fall, that is, 2 m and over 5 m lower than subjects C.M. and J.K., respectively. In this case, skiing shorter radii around the gate was definitely a less efficient solution. From the simultaneous computer video analysis, it can be observed that he performed a higher degree of pivoting with far more side skidding at the beginning of the turn. This is the part where his quality of skiing is far below the standard deviation (see Figure 3b from 14 to 23 m).

Subject C.M. skied completely differently. For all of the time, he was at the top part of the standard deviation interval of the $e_{\text{mec}}$ or even higher. Despite that, the diff($e_{\text{mec}}$) shows his performance in the first 10 inspected meters on the fall line was not better than average (Figure 2b). On the other hand, his skiing starting with the first weight transfer and right before the gate was excellent (Figure 2b). This was mainly due to the faster start of intensive turning at about 15 m on the fall line and the less sharp turning around the gate than, for example, subjects F.C. and J.K. (Figure 4). Nevertheless, his more rounded ski trajectory prolonged the turning section and consequently his bottom part of the turn was virtually less efficient. Making another round turn or an early entrance to the next turn won him some more energy, ending with the highest $e_{\text{mec}}$ of them all (Figure 2a). A similar performance can be seen with U.P. He started with $e_{\text{mec}}$ below the mean (Figure 3a) and ended with almost the highest, far over the standard deviation interval. His skiing was tactically different from that of C.M. He even skied an earlier line right from the beginning (Figure 4). That enabled him far less emphasized pivoting (side skidding) at the beginning of the second turn. There he performed the most efficient skiing (Figure 2b, from 40 to 47 m on the fall line). Both results contradict the beliefs of several ski coaches who teach their racers to ski a more direct line and shorter radii around gates that should result in shorter trajectories and therefore quicker times.

When observing the results of measurements, it sometimes appears that the principle or calculation has an artifact because the $e_{\text{mec}}$ is increasing (Figures 2a and 3a), but it can be mechanically justified through the skier’s muscle work. If the skier crouches and extends in a direction perpendicular to the terrain surface, he adds and takes

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**Table 1  The Point of the Beginning of the Turn and the Point of the End of the Turn on the Fall Line and the Section Time**

<table>
<thead>
<tr>
<th>Racer</th>
<th>Beginning of the turn; fall line (m)</th>
<th>End of the turn; fall line (m)</th>
<th>Time (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>U. Pavlovic</td>
<td>9.22</td>
<td>35.45</td>
<td>3.14</td>
</tr>
<tr>
<td>C. Mayer</td>
<td>10.9</td>
<td>34.38</td>
<td>3.03</td>
</tr>
<tr>
<td>F. Covili</td>
<td>10.3</td>
<td>36.16</td>
<td>3.08</td>
</tr>
<tr>
<td>J. Koblar</td>
<td>13.41</td>
<td>36.43</td>
<td>3.30</td>
</tr>
</tbody>
</table>

*Note: The times are calculated from 0 to 45 m on the fall line because the skiers are skiing there closely toward the fall line and therefore the error is the lowest.*
away potential energy because a component of movement is parallel to gravity, but this does not contribute to the skier’s desirable kinetic energy. If the work of friction and air resistance is less than the increase in potential energy, the energy dissipation is positive. Another detail should be revealed for easier analyzing with \( \text{diff}(e_{\text{mech}}) \). When \( \text{diff}(e_{\text{mech}}) = -g = -9.81 \text{ J/kg·m} \) (see Equations 8 and 9), the kinetic energy and the absolute velocity remain constant or extremum for the kinetic energy and absolute velocity appears.

The measurement examples show some possible uses of the new method. Generally, it can be used for various purposes, ranging from technique to tactics analyses and even ski testing. For example, by using the energy principle, \( \text{diff}(e_{\text{mech}}) \) vs. radius characteristics can also be distinguished. This is useful for estimating a skier’s abilities (technique) or skis’ properties in guiding skis over different skiing radii. It makes analyzing and estimating the quality of skiing far more objective because the way it is calculated eliminates the effects of the different trajectories skied, which would not be the case were a time differentiation to be used. Besides that, it is relatively easy to calculate (Equation 9), it gives a measure of quality at each point of observation, and it is easy to understand because it has a clear mechanical background even though the parameter is pretty complex. The new energy principle clearly shows that time analysis or trajectory analysis can quickly become misleading because achieving a good section time does not necessarily mean a good skiing performance.

The reliability of the results of the introduced quality parameter depends only on the measuring accuracy and consequently the measuring method. Several measuring techniques could be used. However, the new method in conjunction with 3-D kinematical measurements is extremely useful because several other technique parameters can be simultaneously evaluated with a quantitative measure of the quality of skiing (e.g., KinSki 3.1; Supej et al., 2005b). The whole principle could be improved by introducing all segments’ energy contributions, including rotational kinetic energy, but in that case the usage would be mostly limited to 3-D kinematical measurements, where all segments are taken into consideration separately.

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**References**


Supej, M., Kugovnik, O., & Nemec, B. (2004). Kolikšne so razlike v svetovnem pokalu in tekmovalna taktika [How big are the differences at World Cup races and the racing tactics]. *Šport, 52*(4), 75–81.

