An Alternative Model of the Lower Extremity During Locomotion

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An alternative to the Iterative Newton-Euler or linked segment model was developed to compute lower extremity joint moments using the mechanics of the double pendulum. The double pendulum model equations were applied to both the swing and stance phases of locomotion. Both the Iterative Newton-Euler and double pendulum models computed virtually identical joint moment data over the entire stride cycle. The double pendulum equations, however, also included terms for other mechanical factors acting on limb segments, namely hip acceleration and segment angular velocities and accelerations. Thus, the exact manners in which the lower extremity segments interacted with each other could be quantified throughout the gait cycle. The linear acceleration of the hip and the angular acceleration of the thigh played comparable roles to muscular actions during both swing and stance.

The Iterative Newton-Euler or linked segment model is the standard means for computing lower extremity muscle moments. This model sums all segment interactions together as joint reaction forces and therefore cannot be used either for simulation or to establish how the motion of one limb segment affects the other segments. Many researchers have developed models of lower extremity function, but these are generally limited to either swing or stance. For example, Putnam (1991), Chapman, Lonergan, and Caldwell (1984a, 1984b), and Mena, Quanbury, Ross, and Letts (1981) developed swing-phase models because the linked segment model could not be used to examine their stated problems. These models treated the lower extremity as swinging from the hip. Numerous studies also developed models of stance-phase kinematics. By contrast, they generally treated the stance leg as pivoting at the point of contact with the ground. A representative model is the "inverted pendulum" of Mochon and McMahon (1980), which consisted of limb segments supporting a mass representing the trunk.

The purpose of this study was to develop a model applicable to both the swing and stance phases of locomotion that would offer different information than present models. This alternative information includes interactions among the lower extremity segments and their relationship to lower extremity joint moments. This was accomplished by applying the mechanics of the double pendulum to lower extremity motion.

Model Description

A mechanical representation of the double pendulum model is presented in Figure 1. This model makes identical assumptions to the Newton-Euler model (Winter, 1990). That is,
Notation

The subscripts T, L, and F denote the thigh, leg, and foot, respectively, the subscript Lf denotes the leg/foot system, and the subscripts x and y denote coordinate system directions.

- \( m \) = segment mass
- \( I \) = segment moment of inertia about proximal joint
- \( L \) = segment length
- \( d \) = distance from segment mass center to proximal joint
- \( \Theta, \omega, \alpha \) = segment angular position, velocity, and acceleration
- \( a_h \) = acceleration of the hip
- \( g \) = acceleration of gravity
- \( \text{GRF} \) = ground reaction force
- \( \text{CP} \) = center of pressure of ground reaction force
- \( \text{Ankle, Knee, Hip} \) = 2-D locations of joint centers

Figure 1 — Mechanical representation of the double pendulum model.

segment mechanical parameters are assumed constant, joints are frictionless hinges, and the net actions of muscles are represented across each joint individually. In regard to specific considerations of the double pendulum system, the pelvis is treated as a rigid, moving support whether or not the foot is in contact with the ground. In other words, the lower extremity is always suspended from the hip: Ground reaction forces are treated as external forces acting on a swinging double pendulum. For simplicity, it is assumed that the leg and foot are a single system with a constant moment of inertia. This is different from assuming a rigid ankle: The foot is allowed to move with respect to the leg. The underlying assumption is that the motion between the foot and leg is small enough that the moment of inertia of the system varies insignificantly. In practice, the ankle experienced
roughly 30° of motion, but this caused the system moment of inertia to vary by only 2.9% and 3.1% during a single stride of walking and running, respectively.

The model equations for the hip and knee during swing were derived using Lagrange's equation of motion. For the swing phase,

\[ I_L \alpha_L + m_L d_Y_L \left[ L_L \alpha_L \cos(\Theta_L - \Theta_T) + L_T \omega_T^2 \sin(\Theta_L - \Theta_T) ight] + a_{thx} \cos \Theta_L + (a_{thy} + g) \sin \Theta_L] = M_k \]

\[ (I_T + m_L L_T^2) \alpha_T + m_L L_T d_J_L (\alpha_L \cos(\Theta_L - \Theta_T) - \omega_T^2 \sin(\Theta_L - \Theta_T)) + (a_{thx} \cos \Theta_T + (a_{thy} + g) \sin \Theta_T) (m_t d_t + m_{th} L_t) + M_k = M_{th} \]

These are simplified forms of those presented by Putnam (1991). Each term of Equations 1 and 2 is a torque. For example, in Equation 1, the terms containing \( a_T \) and \( v_T^2 \) are, respectively, the torques exerted on the leg by the angular and centripetal accelerations of the thigh. Equation 2, in particular, uses the knee moment on its left-hand side, as opposed to Putnam's corresponding equation, which instead uses the lefthand side of Equation 1.

A formula for ankle moment was derived by determining the linear acceleration of the ankle joint, which was the vector sum of the accelerations of the hip, thigh, and leg. The sum of these accelerations and the force of gravity acting on the foot center of mass exerted a torque about the ankle joint, as shown in Figure 2.

Thus, summing the moments about the ankle yielded a formula for ankle moment during swing:

\[ M_a = I_T \alpha_T + m_T d_F \left[ -a_{thx} \sin \Theta_T + (a_{thy} + g) \cos \Theta_T \\ + L_L [\omega_T^2 \cos(\Theta_T - \Theta_L) - \alpha_L \sin(\Theta_T - \Theta_L)] \\ + L_T [\omega_T^2 \cos(\Theta_T - \Theta_L) - \alpha_T \sin(\Theta_T - \Theta_T)] \right] \]

**Figure 2** — Summation of distal joint accelerations yields the acceleration of the foot.
Equations 1, 2, and 3 are, therefore, alternative means for computing joint moments when the foot is not in contact with the ground. They were extended to stance by adding the influence of external forces under the foot. In contrast to inverted pendulum models, these equations treated the hip joint, not the point of contact with the ground, as the pivot of the swinging system. The moment arms of the ground reaction forces were the perpendicular distances between the center of pressure and each joint. Thus, adding the moment of each ground reaction force about each joint to its respective swing moment equation yielded the following formulas for joint moments during stance:

\[
M_a = M_a \text{ from Equation 3} - \text{GRF}_x(\text{Ankle}_y) - \text{GRF}_y(\text{CP}_x - \text{Ankle}_x)
\]

(4)

\[
M_k = M_k \text{ from Equation 1} - \text{GRF}_x(\text{Knee}_y) - \text{GRF}_y(\text{CP}_x - \text{Knee}_x)
\]

(5)

\[
M_h = M_h \text{ from Equation 2} - \text{GRF}_x(\text{Hip}_y) - \text{GRF}_y(\text{CP}_x - \text{Hip}_x)
\]

(6)

Complete derivations of these equations and all other computational methods used in this paper appear in the appendix.

Methods

One healthy 24-year-old male with a body height of 1.96 m and mass of 82 kg served as the subject for this study. Reflective markers were placed over the greater trochanter, lateral femoral epicondyle, lateral malleolus, and fifth metatarsal head such that they approximated the joint centers. These markers were considered estimates of the true axis of rotation. A polymer-based switch mounted on the heel of the subject was interfaced to a light-emitting diode (LED) to record the instant of heel contact. The subject performed five trials each of walking and running at a self-selected cadence.

Kinematic data were recorded by a 200-Hz NAC high-speed video camera interfaced to a video recorder. The body markers were illuminated by a Motion Analysis MacLite strobing LED light. Planar coordinates of the markers were obtained from the video images using a Motion Analysis VP110 microprocessor interfaced to a SUN minicomputer. One complete stride of running and one of walking were digitized. The coordinates obtained were smoothed using a low-pass Butterworth digital filter with optimal cutoff frequencies obtained from an algorithm based on that of Jackson (1979). The cutoff frequencies ranged from 8 to 12 Hz. Planar coordinates of the video data were used to define the thigh, leg, and foot. Masses, moments of inertia, and center of mass locations of these body segments were calculated using proportions given by Winter (1990).

Segment angles were computed from the marker coordinates. These angles were numerically differentiated twice with a first-order central difference method. The resulting angular positions, velocities, and accelerations were used to compute the linear accelerations of each segment for input to the linked segment equations.

Ground reaction force data were recorded using a floor-mounted AMTI force platform. Force data were sampled at a frequency of 1 kHz. The camera and force data were synchronized using an LED interfaced to the vertical output of the force plate.

Results and Discussion

In the calculation of joint moments, the double pendulum and rigid link methods theoretically produce identical results. Figures 3 and 4 graph ankle, knee, and hip moments during
walking and running, respectively. During walking, the ankle, knee, and hip moments had mean absolute differences between the two models of 0.28 N \cdot m, 1.53 N \cdot m, and 7.63 N \cdot m, respectively. During running, these three moments had mean absolute differences of 0.36 N \cdot m, 3.77 N \cdot m, and 10.16 N \cdot m, respectively, between the two models. These minor differences were due to the various computational methods; the double pendulum model used angular parameters, but the linked segment model also used segment linear accelerations. It was found that the differences between the two models were smallest when linear accelerations were computed from angular parameters. The differences were slightly larger when linear and angular accelerations were separately differentiated from linear and angular positions, demonstrating the intrinsic errors of numerical differentiation. Depending on the method of differentiation, segment linear accelerations were found to differ by as much as 5 m/s² to 8 m/s².

The double pendulum's ankle is neither rigid nor a frictionless pin joint; rather, the leg and foot are modeled as a system with a constant moment of inertia about the knee. Also, the motion of the foot about the ankle is assumed to cause negligible motion at the knee. In other words, the foot is not treated as an independent segment, as are the leg and thigh, because the actions of passive structures about the ankle are relatively large compared to the foot moment of inertia. Clearly, the ankle is not a frictionless pin joint, but the approximation has not prohibited the linked segment model from computing valid data. Likewise, the double pendulum's ankle is a mechanical approximation, and the absolute mean differences of 1.53 N \cdot m and 3.77 N \cdot m for running and walking knee moment data demonstrate its appropriateness.
Figure 4 — Comparison of joint moments computed by double pendulum and linked segment models over the course of one running stride: (a) ankle moment, (b) knee moment, and (c) hip moment. RFC = right foot contact; RTO = right toe-off.

Figure 5 — Moments due to hip acceleration about the hip and knee during (a) one walking cycle and (b) one running cycle. RFC = right foot contact; RTO = right toe-off.
The double pendulum offers additional understanding of human movement because it identifies other factors that influence observed motions. The terms of the double pendulum equations include moments due to the motion of other components of the system in addition to muscles, ground contact, and gravity. Thus, the contributions of segment interactions can be examined by extracting specific terms from the model equations. As an example, it was shown that throughout the walking cycle, linear acceleration of the hip played a substantial role in developing the motion of the thigh and leg (Figure 5). For example, Figure 5a indicates that hip acceleration exerted a 30 N \cdot m hip extension moment near the end of stance; at this time, the hip moment was close to 0 N \cdot m. During running (Figure 5b), a 60 N \cdot m hip moment was exerted by hip acceleration during the middle of swing. As shown in Figure 6, angular acceleration of the thigh segment also played a significant role throughout the gait cycle. In particular, it appeared that the thigh was accelerated forward just prior to swing and at the end of swing to aid knee flexion. These moments were comparable to the knee muscle moments, about 20 N \cdot m during walking and 25 to 35 N \cdot m during running. This suggests that the hip and knee musculature may share the role of knee flexion during locomotion.

A growing body of research demonstrates that the muscles acting on a segment are but one factor influencing its motion (Chapman et al., 1984a; Philips, Roberts, & Huang, 1983; Putnam, 1991). These interactive factors are interesting in their own right, particularly since they appear to be adaptable controls. In other words, certain muscle functions can be substituted with different hip or segment motions to accommodate muscular injury or dysfunction. The exaggerated hip motion of individuals with above-knee amputations (Cappozzo, Figure, Leo, & Marchetti, 1976; Lewallen, Quanbury, Ross, & Letts, 1985) is one such example: Because they lack knee muscles, these individuals employ the motion of the hip and thigh to control the prosthetic leg. Such control adaptations might also be observed in individuals with certain muscular pathologies; for example, a knee extensor dysfunction could cause a number of adaptive gait abnormalities. The systemic approach of the double pendulum could point to the exact problem from among numerous gait abnormalities.

**Conclusion**

For the purpose of joint moment computation, the double pendulum equations yield the same data as the linked segment equations. Therefore, the choice of models for this
purpose is entirely a matter of preference. The double pendulum model, however, offers other kinetic variables related to the interaction of body segments throughout the gait cycle.

References


Appendix

Iterative Newton-Euler Equations for the Calculation of Joint Moments

The equations used for the Iterative Newton-Euler or rigid-link model were adapted from those presented in Winter (1990). In the following equations, Ra, Rk, and Rh represent the reaction forces at the ankle, knee, and hip joint centers, respectively, and a “cm” subscript denotes that the moments of inertia below are about the segment mass center. All other symbols have been defined previously.

**Ankle Moment**

\[ M_a + (\text{GRF}_y \cdot s_1) + (\text{GRF}_x \cdot s_2) - (R_{a_y} \cdot s_3) + (R_{a_x} \cdot s_4) = I_{\text{cm}} \alpha_f \]

where

- \( s_1 \) = center of pressure \(-\) foot center of mass
- \( s_2 \) = foot center of mass \(-\) center of pressure
- \( s_3 = d_r \cos \Theta_f \n
- \( s_4 = d_r \sin \Theta_f \)
Knee Moment

\[ M_k - M_a = (R_{ay} \cdot s_1) - (R_{ax} \cdot s_2) - (R_{ky} \cdot s_3) - (R_{kx} \cdot s_4) = I_{lcm} \alpha_l \]

where \( s_1 = (L_l - d_l) \sin \Theta_l \)
\( s_2 = (L_l - d_l) \cos \Theta_l \)
\( s_3 = d_l \sin \Theta_l \)
\( s_4 = d_l \cos \Theta_l \)

Hip Moment

\[ M_h - M_k = (R_{by} \cdot s_1) - (R_{bx} \cdot s_2) - (R_{hy} \cdot s_3) - (R_{hx} \cdot s_4) = I_{tcm} \alpha_t \]

where \( s_1 = (L_t - d_t) \sin \Theta_t \)
\( s_2 = (L_t - d_t) \cos \Theta_t \)
\( s_3 = d_t \sin \Theta_t \)
\( s_4 = d_t \cos \Theta_t \)

Derivation of Segment Linear Accelerations From Angular Parameters

The following equations apply to the thigh. The equations of the leg and foot take a similar form. The \( x,y \) coordinates of the thigh center of mass can be computed from the position of the hip and the angular position of the thigh segment:

\[ x_t = x_h + d_t \sin \Theta_t \]
\[ y_t = y_h - d_t \cos \Theta_t \]

Doubly differentiating these equations with respect to time yields the linear accelerations of the thigh center of mass.

\[ a_{tx} = a_{hx} + d_t \alpha_h \cos \Theta_t \]
\[ a_{ty} = a_{hy} + d_t \alpha_h \sin \Theta_t \]
\[ -d_t \omega_t^2 \sin \Theta_t \]
\[ -d_t \omega_t^2 \cos \Theta_t \]

Derivation of the Double Pendulum Equations

The basic double pendulum equations were derived using Lagrange's Equation of Motion,

\[ \frac{d}{dt} \frac{\partial KE}{\partial \omega_i} - \frac{\partial KE}{\partial \Theta_i} + \frac{\partial PE}{\partial \Theta_i} = M_i , \]

where the subscript \( i \) denoted either the thigh or leg segment, \( KE \) and \( PE \) were, respectively, the kinetic and potential energies of the system, \( \omega \) and \( \Theta \) were a segment's angular velocity and orientation, respectively, and \( M \) was the external moment applied to a segment.

The kinetic energy of the double pendulum was derived as

\[ KE = \frac{1}{2} (I_{tcm} \omega_t^2 + I_{lcm} \omega_l^2 + m_t v_t^2 + m_l v_l^2) \]  

(1)

where \( v_t \) was the linear velocity of the thigh center of mass, which was the vector sum of the velocities of the hip and of the thigh mass center relative to the hip, that is,
\[ v_T^2 = (v_{ht} + \omega_T d_T \cos \Theta_T)^2 + (v_{hy} + \omega_T d_T \sin \Theta_T)^2 \]

and \( v_{lf} \) was the vector sum of the velocities of the hip, of the knee relative to the hip, and of the foot mass center relative to the knee, that is,

\[ v_{lf}^2 = (v_{ht} + \omega_T L_T \cos \Theta_T + \omega_{l_f} d_{l_f} \cos \Theta_{l_f})^2 + (v_{hy} + \omega_T L_T \sin \Theta_T + \omega_{l_f} d_{l_f} \sin \Theta_{l_f})^2 \]

The potential energy of the double pendulum was derived as

\[ PE = g \left[ (m_T d_T + m_{l_f} L_T)(1 - \cos \Theta_T) + m_{l_f} d_{l_f}(1 - \cos \Theta_{l_f}) \right] + (m_T + m_{l_f}) y_H \tag{2} \]

where \( y_H \) was the vertical position of the hip relative to the ground. The potential energy was a function of any parameter that changed the height of a segment’s mass center. Thus, Equation 2 has terms for the vertical position of the hip and the angular positions of the thigh and leg.

The application of Lagrange’s equation to Equations 1 and 2 yielded for the leg and thigh, respectively,

\[ I_T \alpha_T + m_{l_f} d_{l_f} [L_T \alpha_T \cos (\Theta_L - \Theta_T) + L_T \omega_T^2 \sin (\Theta_L - \Theta_T)] + a_{ht} \cos \Theta_T + (a_{hy} + g) \sin \Theta_T = 0 \tag{3} \]

and

\[ (I_T + m_{l_f} L_T^2) \alpha_T + m_{l_f} L_T d_{l_f} [\alpha_T \cos (\Theta_L - \Theta_T) - \omega_T^2 \sin (\Theta_L - \Theta_T)] + [a_{ht} \cos \Theta_T + (a_{hy} + g) \sin \Theta_T] (m_{l_f} d_{l_f} + m_{l_f} L_T) = 0 \tag{4} \]

Equations 3 and 4 are of the form of the angular case of Newton’s second law of motion: Each term of these equations is either a torque or an inertial term. For example, in Equation 3, the terms containing \( \alpha_T \) and \( \omega_T^2 \) are, respectively, the torques exerted on the leg by the angular and centripetal accelerations of the thigh. The latter terms are the combined effects of hip acceleration and gravity. The right-hand sides of Equations 3 and 4 are equal to zero, denoting that there are no externally applied moments. These equations were extended by adding the net effect of muscle activity to the right-hand side of each equation. The leg segment was acted upon by the knee moment, which was added to the right-hand side of Equation 3 to yield a formula for the knee moment during the swing phase:

\[ I_T \alpha_T + m_{l_f} d_{l_f} [L_T \alpha_T \cos (\Theta_L - \Theta_T) + L_T \omega_T^2 \sin (\Theta_L - \Theta_T)] + a_{ht} \cos \Theta_T + (a_{hy} + g) \sin \Theta_T = \text{Knee Moment.} \tag{5} \]

The thigh segment was acted on by the hip moment on its proximal end and by the negative of the knee moment on its distal end. Therefore, addition of the hip moment to the right-hand side of Equation 4 and the knee moment to its left-hand side yielded a formula for the hip moment during swing:

\[ (I_T + m_{l_f} L_T^2) \alpha_T + m_{l_f} L_T d_{l_f} [\alpha_T \cos (\Theta_L - \Theta_T) - \omega_T^2 \sin (\Theta_L - \Theta_T)] + [a_{ht} \cos \Theta_T + (a_{hy} + g) \sin \Theta_T] (m_{l_f} d_{l_f} + m_{l_f} L_T) + \text{Knee Moment} = \text{Hip Moment.} \]

A formula for ankle moment was derived by determining the linear acceleration of the ankle joint, which was the vector sum of the accelerations of the hip, thigh, and leg.
Thus, summing the moments about the ankle yielded a formula for ankle moment during swing:

\[
\text{Ankle Moment} = I_F \alpha_F + m_F d_F \left\{ -a_{it} \sin \Theta_F + (a_{hy} + g) \cos \Theta_F \right. \\
+ L_L \left[ \omega_L^2 \cos(\Theta_F - \Theta_L) - \alpha_L \sin (\Theta_F - \Theta_L) \right] \\
+ L_T \left[ \omega_T^2 \cos(\Theta_F - \Theta_T) - \alpha_T \sin(\Theta_F - \Theta_T) \right] \right\}
\tag{7}
\]

The double pendulum equations were extended to stance by application of superposition: The lower extremity was still treated as a swinging pendulum but under the additional influence of external forces under the foot.