Dynamics and Energetics of Impacts in Crutch Walking

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The impulsive dynamics associated with the impact of the crutch with the ground is an important topic of research, since this is known to be the main cause of energy loss during crutch gait. In this article, a four-segmental 2D model based on anthropometric body segment parameters is used to analyze various dynamics aspects of such impact. For this purpose, a novel formulation based on the decomposition of the tangent space of the biomechanical system to two subspaces associated with the constrained and admissible motions is developed. Detailed numerical analysis is presented to discuss the effects of body configuration and crutch length on the kinetic energy redistribution, velocity change and impulsive contact forces generated. The conclusions reached via this analysis give guidelines for optimal crutch selection or crutch-use teaching that can be applied to injured subjects. For instance, to reduce energy consumption which leads to a reduction of muscular fatigue.

Keywords: biomechanics, energy analysis, variable topology systems, impact dynamics

Crutch walking is a widespread type of gait among injured people. Crutches are widely used by elderly people, sport-injured people and even paraplegics. In their follow-up study of paraplegic individuals, Jaspers et al. highlighted the psychological and physiological importance for paraplegics to be able to stand and walk on their own, which can be achieved by means of crutch walking (Jaspers et al., 1997). The main problem of crutch locomotion is that it is much more demanding for the subject than normal walking.

It was shown in (Noreau et al., 1995) that crutch walking is highly demanding for the upper-body muscles which are not suited for such efforts. The authors found that the preferred speed and cadence in crutch walking remain at about 50% of those in normal walking. Furthermore, the energy consumption of crutch walking has been reported to be much higher than that of normal walking in terms of metabolic cost (Thys et al., 1996; Waters & Lunsford, 1985). This excess effort can cause health problems in crutch users, for example, upper-limb joint degeneration (Opila et al., 1987), or even injuries in the nervous system like bilateral ulnar neuropatia (Malkan, 1992). Both the high energy demand and the health risk associated with crutch walking led scientists to develop dynamic analyses to better understand this type of motion. In (Shoup et al., 1974), the authors measured joint displacements in the sagittal and frontal planes during crutch walking and compared their results with existing experimental data based on normal walking. Based on this work, the authors concluded that crutch design should try to minimize the vertical motion of the body and the impact associated to the crutch tip planting to make the motion less demanding.

Other works analyze the dynamic consequences of different crutch designs, for example standard and spring-loaded crutches. It was shown in (Segura & Piazza, 2007) that the ground reaction impulse magnitude is substantially reduced (about 25%) when spring-loaded crutches are used. Nevertheless, compared with regular rigid crutches, no significant difference has been found in terms of energetic cost employing heart rate-based measurements (LeBlanc et al., 1993). Similar conclusions were drawn regarding rocker-bottom axillary crutches (Nielsen et al., 1990). The effect of the crutch length was also analyzed previously and no significant influence was found neither on the applied forces and moments (Reisman et al., 1985), nor on the oxygen consumption of the subjects (Mullis & Dent, 2000).

It is important to highlight that most of the works that appear in the literature regarding crutch walking, like the ones cited previously, investigate the kinematics and kinetics of the motion based on direct measurements from human subjects (joint displacements, forces, energy consumption). These studies provide useful information regarding the particular motion of the analyzed subjects and allow to draw conclusions based on the measured data. However, based on then it is difficult to analyze for example the effects of the body posture or other parameters on the required forces and moments since each subject walks in a particular way.
In this article we use a different approach, an anthropometric-based four-segmental model of the human body is presented to analyze the effect of the body configuration and the crutch length on different aspects of the dynamics associated with the impact of the crutch with the ground at the end of foot stance. Namely, the kinetic energy redistribution, the energy loss per unit distance, the foot separation velocity after impact, and the contact impulses generated on the crutch. We focus on this particular aspect of the crutch gait because it is one of the major causes of energy loss during motion (Shoup et al., 1974), similar to the case of heel-strike in normal human walking (Kuo, 2002). Model-based approaches have been widely used to better understand normal human walking, and have provided useful insight regarding the mechanical principles underlying human locomotion (Donelan et al., 2002; Kuo et al., 2005; Adamczyk et al., 2006). In (Van der Spek et al., 2003) a model-based approach was presented to analyze the static stability of crutch-supported paraplegic standing for different values of hip-joint stiffness and crutch-to-feet distance. Nevertheless, as far as the authors know, little research has been done on model-based studies of the dynamics of the different phases of crutch locomotion (i.e., crutch-stance phase, foot impact, foot-stance phase, crutch impact).

The event of crutch impact represents a change in the topology of the multibody system since some physical constraints are added (crutch-ground contact) and other become passive (foot-ground contact). Based on the Jacobian matrix characterizing the new constraints acting on the system, a concept that allows a complete decoupling of the impulse-momentum level dynamics equations and the kinetic energy of the subject is presented. This is obtained via a physically meaningful decomposition of the tangent space of the system to two mutually independent and orthogonal subspaces associated with the constrained and admissible motions.

### Methods

The dynamic model of the subject has four segments, and its general configuration in the sagittal plane is represented by six coordinates which form the vector of generalized coordinates \( \mathbf{q} = [x, y, \theta_1, \theta_2, \theta_3, \theta_4]^T \) (Figure 1). Coordinates \( x \) and \( y \) indicate the position of point A (representing the foot) with respect to the absolute inertial frame. Angle \( \theta_1 \) indicates the absolute orientation of the leg with respect to the vertical, and \( \theta_2, \theta_3, \) and \( \theta_4 \) denote the relative angles of the torso with respect to the leg (hip angle), the arm with respect to the torso (shoulder angle), and the forearm and crutch with respect to the arm (elbow angle). The defined angles \( \theta_i \) \( (i = 1,2,3,4) \) are measured positive in the clockwise direction, as indicated in Figure 1.

Points B, C, D and E represent the hip joint, the shoulder joint, the elbow joint and the tip of the crutch, respectively. We assume that each segment is a rigid body and that the forearm and the crutch form a single segment. The length, mass, moment of inertia and center of mass of each segment are respectively denoted by \( l_i, m_i, I_i \) and \( G_i, i = 1,2,3,4 \) (1-leg, 2-torso, 3-arm, 4-forearm and crutch). (These segments will be named in singular throughout the text of this article. However, note that segment “1” includes the two legs, segment “3” includes the two arms, and segment “4” includes the two forearms and crutches.) Parameter \( a_i \) represents the position of the center of mass (Figure 1). It is important to note that some possible values of \( q \) are not physically meaningful. Hence, during the analysis we will impose the following conditions:

\[
\begin{align*}
\theta_4 \geq 0 \quad \text{and} \quad \theta_1 + \theta_2 \geq 0 \\
90^\circ < \theta_1 + \theta_2 \leq 180^\circ \\
270^\circ < \theta_4 \leq 360^\circ
\end{align*}
\]

which impose that the leg and the torso are inclined forward at crutch impact (1), the arm is directed forward (2), and the elbow is bent in a normal way (3).

The lengths and inertial parameters of the body segments are obtained from anthropometric data provided in (Nikolova & Toshev, 2007). These are summarized in Table 1. The values for segment 4 (forearm and crutch) are obtained taking into account that the conventional crutch length for the subject analyzed is \( L_c = 103.4 \) cm and a typical value for the mass of the crutch is 0.5 kg, these values have been taken from the website of crutch manufacturers (http://www.convaquip.com/Crutches/Crutch-Torso/Crutch-Crutch-c22/).

The configuration of the considered biomechanical system can be described by the \( 6 \times 1 \) dimensional array \( \mathbf{q} \) which was previously given. The time derivatives of these coordinates give a possible set of generalized velocities \( \dot{\mathbf{q}} \). Based on this, the kinetic energy of the system can be expressed as

\[
T = \frac{1}{2} \sum_{i=1}^{4} m_i \dot{x}_i^2 + \frac{1}{2} \sum_{i=1}^{4} I_i \dot{\theta}_i^2 = \frac{1}{2} \dot{\mathbf{q}}^T \mathbf{M}(\mathbf{q}) \dot{\mathbf{q}}
\] (4)

### Table 1 Anthropometric body segment parameters of the four-segmental model

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Leg (( i = 1 ))</th>
<th>Torso (( i = 2 ))</th>
<th>Arm (( i = 3 ))</th>
<th>Forearm and Crutch (( i = 4 ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>( m_i )</td>
<td>28.6 kg</td>
<td>34.4 kg</td>
<td>4.2 kg</td>
<td>3.4 kg</td>
</tr>
<tr>
<td>( I_i )</td>
<td>6,930 kg·cm(^2)</td>
<td>11,672 kg·cm(^2)</td>
<td>441.6 kg·cm(^2)</td>
<td>4,026 kg·cm(^2)</td>
</tr>
<tr>
<td>( l_i )</td>
<td>88.2 cm</td>
<td>68.2 cm</td>
<td>30.9 cm</td>
<td>128.1 cm</td>
</tr>
<tr>
<td>( a_i )</td>
<td>56.4 cm</td>
<td>38.1 cm</td>
<td>14.8 cm</td>
<td>30.1 cm</td>
</tr>
</tbody>
</table>
where \( \mathbf{v}_{ci} \) \((i = 1,2,3,4)\) denotes the velocity of the centers of mass, \( \omega = \sum_{i} \dot{q}_{i} \) is the absolute angular velocity of the \( i \)th segment, and \( \mathbf{M} \) is the mass matrix of the system. The dynamics equations can generally be written as

\[
\mathbf{M} \ddot{\mathbf{q}} + \mathbf{c}(\mathbf{q}, \dot{\mathbf{q}}) = \mathbf{f}_{A} + \mathbf{f}_{G} \quad (5)
\]

where \( \mathbf{c} \) represents the Coriolis and centrifugal effects, and \( \mathbf{f}_{A} \) and \( \mathbf{f}_{G} \) are respectively the generalized applied and constraint forces. The applied forces are associated with joint torques produced by the muscles and also gravity, and the constraint forces arise when physical contact constraints appear either on the foot or on the tip of the crutch.

We focus our study on the impulsive event associated with the impact of the crutch tip with the ground. Let us consider that \( t_{i} \) represents the time instant when this impact takes place and the topology of the system changes since new constraints are suddenly imposed. This event takes place in the time interval \([t_{i},t_{i}']\) where \( t_{i}' \) and \( t_{i}' \) denote the so-called pre- and postimpact instants. The duration of this interval is generally considered to be very short compared with the characteristic time scale of the finite motion of the system. Therefore, in \([t_{i},t_{i}']\) the configuration of the system is assumed to be constant and the dynamics can be modeled at the impulse-momentum level. It is important to note that at time \( t_{i} \) the configuration must satisfy the following relationship

\[
I_{c} \cos(\theta_{i}) + I_{z} \cos(\theta_{i} + \theta_{z}) + I_{l} \cos(\theta_{i} + \theta_{z} + \theta_{l}) + I_{l} \cos(\theta_{i} + \theta_{z} + \theta_{l} + \theta_{t}) = 0 \quad (6)
\]

since both the foot (point A) and the crutch tip (point E) are horizontally aligned. The impulsive event that occurs at crutch impact is characterized by inert constraints (Pars, 1965; Kövecses and Cleghorn, 2003), which impose that the postimpact velocity of point E, \( \mathbf{v}_{E}^{+} \), is zero. They can generally be written as

\[
\mathbf{v}_{E}^{+} = \mathbf{A} \mathbf{q}_{+}^{+} = 0 \quad (7)
\]

where \( \mathbf{q}_{+}^{+} \) stands for \( \mathbf{q} \) at \( t_{i}^{+} \), and \( \mathbf{A} \) is the \( 2 \times 6 \) dimensional constraint Jacobian matrix. The expression of this matrix is given in the appendix of this article. Inert constraints can be considered as impulsive, nonholonomic constraints and they represent the required topology at the postimpact instant; i.e., the tip of the crutch be in contact with the ground without slipping.

The tangent space associated with the dynamic model of the biomechanical system can be seen as a six-dimensional linear space interpreted for each configuration (Blajer, 1997; Kövecses, 2008). A single representation of this can be used for the whole impact interval \([t_{i},t_{i}']\) since the configuration is assumed to be unchanged. The Jacobian of the impulsive constraints in Eq. (7) can be employed to decompose the tangent space to the space of constrained motion (SCM) and the space of admissible motion (SAM) for both the pre- and postimpact instants. Such subspaces can be defined so that they are orthogonal to each other with respect to the natural, mass metric of the tangent space (Kövecses, 2008). The decomposition of kinematic and kinetic quantities to these subspaces can be accomplished via two projector operators (Kövecses, 2008; Kövecses & Piedboeuf, 2003). The projector associated with the SCM can be written as

\[
\mathbf{P}_{c} = \mathbf{M}^{-\frac{1}{2}} \mathbf{A}^{T} \left( \mathbf{A} \mathbf{M}^{-\frac{1}{2}} \right)^{-1} \mathbf{A} \quad (8)
\]

and the projector for the SAM can be obtained as

\[
\mathbf{P}_{s} = \mathbf{I} - \mathbf{P}_{c} = \mathbf{I} - \mathbf{M}^{-\frac{1}{2}} \mathbf{A}^{T} \left( \mathbf{A} \mathbf{M}^{-\frac{1}{2}} \right)^{-1} \mathbf{A} \quad (9)
\]

where \( \mathbf{I} \) is the \( 6 \times 6 \) identity matrix. The detailed mathematical derivation to obtain the expressions above is outlined in (Kövecses, 2008).

Using the above operators, the generalized velocities \( \mathbf{q} \) can be decomposed as

\[
\mathbf{q} = \mathbf{v}_{c} + \mathbf{v} = \mathbf{P}_{c} \mathbf{q} + \mathbf{P}_{s} \mathbf{q} \quad (10)
\]

where \( \mathbf{v}_{c} = \mathbf{P}_{c} \mathbf{q} \) and \( \mathbf{v} = \mathbf{P}_{s} \mathbf{q} \) represent the two components associated with the SCM and SAM, respectively. The given projector operators can also be used to decompose any vector of generalized forces \( \mathbf{f} \) or generalized impulses \( \mathbf{I} \). For this case the transpose of the operators given above has to be used (Kövecses, 2008; Modarres Najafabadi, 2008):

\[
\mathbf{f} = \mathbf{f}_{c} + \mathbf{f} = \mathbf{P}_{c} \mathbf{f} + \mathbf{P}_{s} \mathbf{f} \quad \text{and} \quad \mathbf{I} = \mathbf{I}_{c} + \mathbf{I} = \mathbf{P}_{c} \mathbf{I} + \mathbf{P}_{s} \mathbf{I} \quad (11)
\]

where \( \mathbf{f}_{c} = \mathbf{P}_{c} \mathbf{f} \) and \( \mathbf{I}_{c} = \mathbf{P}_{c} \mathbf{I} \) are components associated with the SCM, and \( \mathbf{f}_{s} = \mathbf{P}_{s} \mathbf{f} \) and \( \mathbf{I}_{s} = \mathbf{P}_{s} \mathbf{I} \) are their counterparts contained in the SAM.

The dynamics of the impulsive motion associated with the impact of the crutch tip with the ground can be characterized by impulse-momentum level dynamics equations. Based on Eqs. (4) and (5), these can be obtained in a general form as (Bahar, 1994; Kövecses and Cleghorn, 2003)

\[
\left[ \frac{\partial \mathbf{T}}{\partial \mathbf{q}} \right] = \mathbf{I}_{c} + \mathbf{I}_{s} \quad (12)
\]

where “–” and “+” denote the pre- and postimpact instants, \( \mathbf{I}_{c} \) and \( \mathbf{I}_{s} \) are the impulses of the generalized applied and constraint forces, and \( \left[ \frac{\partial \mathbf{T}}{\partial \mathbf{q}} \right] = -\mathbf{I}_{c} \) is the negative of the impulse of the generalized inertial forces. Since the applied muscle forces are assumed to be finite, then for this case we have \( \mathbf{I}_{c} = 0 \). The impulsive constraint forces are caused by the inert constraints in Eq. (7), which are responsible for the sudden change of topology of the system, hence,

\[
\mathbf{I}_{s} = \mathbf{A}^{T} \mathbf{\lambda} \quad (13)
\]

where \( \mathbf{\lambda} \) denotes the impulse of the constraint forces. Equation (13) is valid when the foot lifts up after the crutch impact (i.e., when \( \dot{y} > 0 \)), since in that case the
only external impulses appear on the tip of the crutch. Should the impact event cause a situation that \( y^* = 0 \), then this would mean that the foot stays in contact with the ground at time \( t_i^* \). In such a case, a noninstantaneous double support phase follows the impact and impulses on the foot can also appear during the interval \( [t_i^*, t_f^*] \). This situation is not desirable since the gait would not evolve in a natural way. In this paper, we consider impacts that satisfy the condition \( y^* > 0 \), and therefore Eq. (13) holds.

Using Eq. (10) and taking into account the orthogonality of projectors \( \mathbf{P}_s \) and \( \mathbf{P}_a \) with respect to \( \mathbf{M} \), it can be shown that the kinetic energy can also be completely decoupled as

\[
T = T_c + T_a = \frac{1}{2} v_c^T \mathbf{M}_c v_c + \frac{1}{2} v_a^T \mathbf{M}_a v_a \tag{14}
\]

Any force or impulse arising in the SCM will change only \( T_c \) leaving \( T_a \) unaffected. In addition, any variation of the generalized velocities that influences \( v_c \) or \( v_a \) only will cause a change in \( T_c \) or \( T_a \), respectively, while leaving the other unchanged. The impulsive event characterized by \( \mathbf{A} \) (with the assumption of ideal constraint realization) gives rise to forces and impulses which will influence \( T_c \) only.

Based on the decompositions detailed previously it can also be seen that the impulsive dynamics equations in (12) can also be decoupled into the equations associated with the SCM

\[
\begin{bmatrix} \frac{\partial T_c}{\partial \dot{v}_c} \end{bmatrix} = \mathbf{M} (v^*_c - v_c^-) = \mathbf{A}^T \mathbf{\lambda} \tag{15}
\]

and the equations associated with the SAM

\[
\begin{bmatrix} \frac{\partial T_a}{\partial \dot{v}_a} \end{bmatrix} = \mathbf{M} (v^*_a - v_a^-) = 0 \tag{16}
\]

From Eq. (16), and taking into account that \( \mathbf{M} \) is invertible, it is immediately visible that \( v^*_a = v_a^- \) and thus, \( T_a^* = T_a^- \). Based on Eqs. (7), (10) and (15), we can conclude that \( v^*_c = 0 \), and therefore Eq. (15) turns into

\[
-\mathbf{M} v_c^- = \mathbf{A}^T \mathbf{\lambda} \tag{17}
\]

Also, using that \( v^*_a = 0 \), we have that at the postimpact instant

\[
\dot{\mathbf{q}}^* = \mathbf{v}_a^* = v_a^- = \mathbf{P}_a \mathbf{q}^- \tag{18}
\]

which gives the solution for the postimpact generalized velocities \( \dot{\mathbf{q}}^* \) of the system.

In terms of energetics, it can be observed that the postimpact kinetic energy of the SCM \( T_c^* = 0 \) (because \( \mathbf{v}_c^* = 0 \)). In addition, the fact that \( v^*_a = v_a^- \) implies that \( T_a^* = T_a^- \). Hence, we can say that \( T^* = T_c^- + T_a^- \), i.e., the preimpact kinetic energy of the SAM is conserved whereas \( T^* \) is completely lost at impact. This justifies the importance of the kinetic energy decomposition introduced in Eq. (14) for the preimpact instant because this indicates the part of the total preimpact energy being lost \( (T_c^-) \) and the one that stays in the system after impact \( (T_a^-) \).

Based on (10), (14), (17) and using that \( v_c^T \mathbf{M}_c v_c = 0 \), it is possible to write that

\[
-(\mathbf{q}^-)^T \mathbf{M}_c v_c^- = -2T_c^- = (\mathbf{q}^-)^T \mathbf{A}^T \mathbf{\lambda} = (\mathbf{v}_c^-)^T \mathbf{\lambda} \tag{19}
\]

which shows that for a given preimpact velocity, the impulse of the constraint forces is proportional to \( T_c^- \). This can make it possible to express an explicit relationship between the impulses generated by the topology change \( \mathbf{\lambda} \) and the kinetic energy content that is lost in the event \( T_c^- \). The magnitude of the impulses can be calculated directly from (17) which yields

\[
\mathbf{\lambda} = -(\mathbf{A}^T \mathbf{\lambda})^{-1} \mathbf{A} v_c^- \tag{20}
\]

The constraint impulses \( \mathbf{\lambda} \) appear to set the velocity of the crutch tip to zero according to (7). Therefore, the collision is supposed to be perfectly inelastic, which appears to be a reasonable and widely used assumption when analyzing walking systems. The second component of \( \mathbf{\lambda} = [\mathbf{\lambda}_c, \mathbf{\lambda}_a] \) is the impulse perpendicular to the ground. The first component of \( \mathbf{\lambda} \), the horizontal impulse, can be more complex in nature, as it can come either from friction or from tangential deformation of the bodies in contact.

Concerning the energetics, we will look at how the energy loss per impact \( (T_c^-) \) and the energy loss per unit distance covered during foot stance \( (\mathbf{\xi} = T_c^- / L_y \) \), where \( L_y \) is the foot to crutch distance at impact) are affected by the subject’s posture and the crutch length. We will also address the variations of the vertical component of the foot postimpact velocity \( (y^*) \) and that of the impulse of the constraint forces \( (\mathbf{\lambda}) \). The foot to crutch distance \( L_y \) can be determined as a function of \( \mathbf{q} \) as

\[
L_y (\mathbf{q}) = l_1 \sin(\theta_1) + l_2 \sin(\theta_1 + \theta_2) + l_3 \sin(\theta_1 + \theta_2 + \theta_3 + \theta_4) \tag{21}
\]

We assume that just before the impact the foot is on the ground (i.e., \( y = 0 \)) and that the crutch tip (point E) collides the ground. Given \( \theta_1 \) and \( \theta_2 \), there are several physically feasible configurations which lead to different values of the quantities examined. This is why we also impose conditions on the crutch-ground angle \( \varphi \) (Figure 1). Angles \( \theta_2 \) and \( \theta_3 \) are then obtained using Eq. (6) and the following relationship

\[
\theta_1 + \theta_2 + \theta_3 + \theta_4 - \varphi = 450^\circ \tag{22}
\]

Angles \( \theta_1 \) imposes in the analysis are chosen according to kinematic studies of subjects walking with crutches (Noreau et al., 1995; Shoup et al., 1974). Concerning preimpact velocities, these studies also show that relative angular velocities associated with the hip, shoulder and elbow joints are approximately zero before impact (Noreau et al., 1995; Shoup et al., 1974). Then, based on these works, we assume that \( \dot{\theta}_1 = \dot{\theta}_2 = \dot{\theta}_4 = 0 \) . The angular velocity \( \dot{\theta}_1 \) can take various values depending
on the walking speed of the particular subject which may depend on the specific pathology. For this angular velocity, we assumed a value of $\dot{\theta}_i = 1 \text{ rad/s}$.

**Results**

As stated in the introduction of this article, the presented modeling approach is used to analyze the effects of the **crutch length** and **body configuration** at crutch impact on the kinetic energy redistribution, the energy loss per unit distance, the foot separation velocity, and the impulsive contact forces generated on the crutch tip. The crutch distance, the foot separation velocity, and the impulsive energy loss at impact is minimized (minimum value for the smallest angle $\theta_i$) when the body inclination $\theta_i + \theta_2$ (Figure 3). The maximum value of $\xi$ does not depend significantly on the body inclination either, and a long crutch tends to reduce its magnitude. Therefore, a long crutch seems to be a good choice to minimize losses over the distance covered during the step.

An important aspect of the impulsive dynamics of crutch impact is the foot separation from the ground after impact. This is measured by velocity $\dot{y}^+$ at the postimpact time, which can be calculated using Eq. (18). An impact with a high postimpact normal velocity of the foot will result in a reduction of the push-off effort required to lift up the body (Shoup et al., 1974).

None of the impacts investigated yields a negative value for $\dot{y}^+$, which means that for all the configurations studied the foot tends to lift up after collision. It can also be observed that the higher $\theta_i + \theta_2$ is, the higher the maximum value of $\dot{y}^+$ is (Figure 4). For $\theta_i + \theta_2 = 25^\circ$, this maximum value does not vary significantly with the length of the crutch. However, for the considered configurations, the minimum value and the mean value of $\dot{y}^+$ are higher for a shorter crutch (Figure 4).

In crutch impacts, the magnitude of the normal impulse is in general larger than the component of the tangential impulse (Figures 5 and 6). Note that, even though the crutch tip has no tangential velocity at impact (due to the choice of $\dot{q}$), a tangential component of the impulse $\vec{I}_x$ is obtained to prevent the crutch tip from sliding forward. (The tangential component is expressed in absolute value in Figure 5; however, the analysis gave negative values for $\vec{I}_x$. Since the tangential impulse is directed toward the negative sense of the x-axis, this means that the crutch tip would tend to slip forward.) The normal component of the impulse $\vec{I}_n$ appears to set the normal velocity of the crutch tip to zero.

So far, the relative torso angle $\theta_i$ was kept constant and equal to zero, i.e., the torso was supposed to be aligned with the legs for all impacts. We analyze now the effects of this angle together with $\theta_i$ on $T_c$, $\xi$, $\dot{y}$, and the norm of the contact impulse $\|\vec{I}\|$. We keep here the crutch length constant and equal to its recommended value $L_0$. The values of angle $\theta_i$ range from $-10^\circ$ to $20^\circ$. We note that negative values of $\theta_i$ may physically be possible, since some paraplegic subjects walk with a
Figure 2 — Kinetic energy lost at impact $T_c$ as a function of $\Delta L$ for different body inclinations $\theta_1 + \theta_2$ between 10° and 25° (The torso relative angle is considered $\theta_2 = 0°$).

Figure 3 — Energy loss per unit distance $\xi$ as a function of $\Delta L$ for different body inclinations $\theta_1 + \theta_2$ between 10° and 25° (The torso relative angle is considered $\theta_2 = 0°$).
Figure 4 — Postimpact normal velocity of the foot $\dot{y}$ as a function of $\Delta L$ for different body inclinations $\theta_1 + \theta_2$ between 10° and 25° (The torso relative angle is considered $\theta_2 = 0^\circ$).

Figure 5 — Tangential component of $\vec{F}$ as a function of $\Delta L$ for different body inclinations $\theta_1 + \theta_2$ between 10° and 25° (The torso relative angle is considered $\theta_2 = 0^\circ$).
negative torso angle according to (Noreau et al., 1995; Shoup et al., 1974). The results of this study are shown in Figure 7. Note that for given angles $\theta_1$ and $\theta_2$, there is a range of possible configurations of the subject (depending on the angle $\varphi$ that the subject chooses) and, therefore, a range of possible $T_c^{-}$, $\xi$, $\gamma^*$, and $\|\mathbf{F}\|$ for each pair of angles. The plots show their mean values as functions of $\theta_1$ and $\theta_2$.

A large torso angle minimizes the energy loss per distance covered during the foot stance phase and the norm of the impulse of the constraint force developed (Figure 7). It can be seen in the same figure that higher postimpact velocities of separation of the foot are obtained for small positive or negative angles of the torso.

**Discussion**

In this article we derived the impulsive dynamics equations for a four-segmental 2D model in the sagittal plane of a subject walking with elbow crutches to analyze crutch impacts. Anthropometric body segment parameters and real crutch parameters were used to develop the model. Based on the Jacobian of the impulsive constraints, we introduced a method that allows a complete decomposition of the dynamics equations and the kinetic energy of the biomechanical system to the spaces of constrained and admissible motions. By means of this approach we derived expressions to determine different dynamic performance indicators associated with the crutch impact, such as the kinetic energy loss, the contact impulses developed and the postimpact velocity of foot separation.

Numerical simulations have been performed to analyze the effects of the crutch length and the body configuration on the above performance indicators. From the obtained results several guidelines can be established to improve crutch locomotion, e.g., to minimize energetic losses at impacts, to minimize the magnitude of the contact impulses which may damage the joints, or to facilitate the separation of the foot after impact. These results can be of interest in various applications, e.g., in rehabilitation and in crutch design. It must be highlighted that modeling approaches, such as the one used in this paper, are well suited to studying crutch gait due to the high variability seen across crutch walkers. In the following we provide a discussion of the main results obtained in the analysis.

Regarding the energy loss at one single impact, which is accounted for by the term $T_c^{-}$, this is minimized if the subject walks keeping the body straight ($\theta_1 = 10^\circ$, $\theta_2 = 0^\circ$) and with the minimum angle $\varphi$ possible. In such a case, all the crutch lengths lead to similar values of minimum energy loss (Figure 2). It has to be pointed
out that minimizing $T_c^-$ is good in crutch walking to reduce the muscular fatigue of the subject, since such energetic losses must be compensated later during the walking cycle by the muscles.

Based on the results related to $T_c^-$ it can also be concluded that a low value of $\Delta L$ (shorter crutch) yields a smaller range of feasible $T_c^-$. Thus, for a non-expert subject that cannot control properly the angle $\varphi$ that minimizes $T_c^-$ it may be better to select a shorter crutch, since in such a case the possible variations of the energy loss at impact is low. Conversely, if the subject is experienced enough to control angle $\varphi$ to minimize the energy loss, then the selection of the length of the crutch is not a critical aspect.

Regarding the energy loss per unit distance $\xi = T_c^- / L_s$, a crutch longer than the conventional is advisable to minimize this performance indicator. This loss does not depend significantly on the body inclination. Note that based on the analysis of $T_c^-$ only, the selection of a long crutch was not advisable because the maximum energy loss grew with $\Delta L$ (Figure 2). However, the maximum value of $\xi$ decreases with $\Delta L$. This is because a long crutch provides longer steps (longer distance $L_s$) and thus the energy loss per distance covered during foot-stance is reduced.

The results presented here regarding the influence of the crutch and the body posture on the energetics of the impact are new, the authors did not find other studies analyzing this aspect of crutch walking. Other studies, such as (Mullis & Dent, 2000; Kawashima et al., 2006) included measurements of the energetic cost of transport based on oxygen consumption for real subjects walking with crutches. There is a relationship between the oxygen consumption and the effective muscular work, which mainly compensates for the energetic loss at impacts. It has to be pointed out that the values for the cost of transport (in J kg$^{-1}$ m$^{-1}$) given in (Mullis & Dent, 2000; Kawashima et al., 2006) are approximately the same order of magnitude as the ones presented in the Results section taking into account that the total mass of our model is 70.6 kg (Table 1).

The separation of the foot from the ground at post-impact time is easier when walking with a crutch shorter than the conventional, since velocity $\dot{y}^+$ is higher. Therefore, it is advisable to select a short crutch for a patient that has difficulties in lifting up the feet after impact, thus reducing push-off effort. The postimpact normal velocity of the foot $\dot{y}^+$ also grows with $\theta_1 + \theta_2$, so that, leaning the body forward may also facilitate push-off separation.

The magnitude of the contact impulses does not depend significantly on the body inclination, but it does depend on the crutch length. It can be clearly concluded that the use of a long crutch is better to minimize the normal impulse $\lambda_y$. However, one can observe that a long crutch provides, in general, larger values of $\lambda_y$, which is not desirable because then the risk of sliding forward is higher. Large values of these impulses are also not desired because they are transmitted to the joints and may cause

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**Figure 7** — Influence of the leg inclination $\theta_1$ and the torso angle $\theta_2$ on the mean values of $T_c^-$, $\xi$, $\dot{y}^+$ and $\|\lambda\|$ (Crutch length is equal to the recommended $L_0$).
fatigue or joint damage as it was pointed out in (Segura & Piazza, 2007; Shoup et al., 1974).

A correlation can be observed among Figures 3, 4 and 6, showing the results for $\xi$, $y'$, and $\dot{\lambda}$, respectively. Hence, less efficient impacts in terms of energy expenditure per unit distance are the ones that produce higher separation velocities $y'$ of the foot. According to this, it can be concluded that there is a trade-off depending on which quantity we want to optimize, i.e., the minimization of $\xi$ and the maximization of $y'$ cannot be achieved at the same time. It can be also seen that higher energy loss per unit distance has a direct correlation with the magnitude of the normal impulse. This was expected because, in fact, energy is lost due to the negative work done by the impulses acting on the tip of the crutch. In addition, the normal impulse is correlated with the separation velocity of the foot after impact, $y'$. This relationship makes sense because the normal impulse causes a sudden change of velocity of the subject’s center of mass (COM). Then, large normal impulses cause more velocity change of the COM which facilitates the separation of the foot.

The torso angle $\theta_2$ has also an effect on the dynamics characteristics. A torso leaning forward with respect to the leg ($\theta_2 > 0$) yields less energy consumption per unit distance and reduces the magnitude of the contact impulses. Conversely, if the torso is in a straight position or leans slightly to the back, the foot lifts up easier from the ground. A good compromise can be achieved by a torso relative angle $\theta_2 = 10^\circ$, since this angle provides good energetic efficiency of the impact, a relatively low magnitude of the impulse, and a relatively high velocity $y'$.

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**References**


Appendix

Expression of the Jacobian Matrix $A$

For the biomechanical model of the subject with crutches (Figure 1), the Jacobian matrix of the inert constraints in Eq. (7) has the following expression:

$$A = \begin{bmatrix}
A_{11} & A_{12} & A_{13} & A_{14} & A_{15} & A_{16} \\
A_{21} & A_{22} & A_{23} & A_{24} & A_{25} & A_{26}
\end{bmatrix}$$

where its 12 elements can be obtained as

\begin{align*}
A_{11} &= 1, \\
A_{12} &= 0, \\
A_{13} &= l_1 \cos(\theta_1) + A_{14}, \\
A_{14} &= l_2 \cos(\theta_1 + \theta_2) + A_{15}, \\
A_{15} &= l_3 \cos(\theta_1 + \theta_2 + \theta_3) + A_{16}, \\
A_{16} &= l_4 \cos(\theta_1 + \theta_2 + \theta_3 + \theta_4), \\
A_{21} &= 0, \\
A_{22} &= 1, \\
A_{23} &= -l_1 \sin(\theta_1) + A_{24}, \\
A_{24} &= -l_2 \sin(\theta_1 + \theta_2) + A_{25}, \\
A_{25} &= -l_3 \sin(\theta_1 + \theta_2 + \theta_3) + A_{26}, \\
A_{26} &= -l_4 \sin(\theta_1 + \theta_2 + \theta_3 + \theta_4).
\end{align*}