Three-Dimensional Interactions in a Two-Segment Kinetic Chain. Part I: General Model

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The motion of a body segment is determined by joint torques and by the motions of the segments proximal or distal to it. This paper describes a three-dimensional model that was used to determine the effects of the shoulder and elbow joint torques and of the upper trunk and arm motions on the angular accelerations of the arm segments during baseball pitching. Equations were developed to fractionate the three-dimensional components of the angular acceleration vector of each segment into angular acceleration terms associated with the joint torques made on the segment, and into various "motion-dependent" angular acceleration terms associated with the kinematic variables of the arm segments. Analysis of the values of the various motion-dependent angular acceleration terms permitted the determination of their contributions to the motion of the segment. Although the model was developed to provide further understanding of the mechanics of the throwing arm during baseball pitching, it can be used to analyze any two-segment two-dimensional or three-dimensional motion.

A full understanding of the cause-effect mechanisms that produce any body motion requires the use of a model to link the kinematics of the motion with the kinetic factors responsible for it. This can be achieved in part by using an inverse dynamics approach to compute the net joint forces and torques exerted on a segment by the segments adjacent to it (Andrews, 1974, 1982, 1983). The joint torque reflects the muscular activity at a given joint, except near the limits of the range of motion, where bones, ligaments, and other passive structures also may contribute to it (Andrews, 1982). Thus the inverse dynamics approach can help to link the motions of a segment with the muscular activities at its articulations. However, the motions of the segment also are affected by the joint forces, and these are in turn associated with the motions of the adjacent segments. Therefore it

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Nomenclature

- \( a_{Gd}, a_{Gu} \) Linear accelerations of Gd and Gu, respectively
- \( a_s \) Linear acceleration of the right (throwing) shoulder
- \( F_D \) Force exerted on the distal segment at the elbow
- \( F_U \) Force exerted on the upper arm at the shoulder
- \( g \) Acceleration of gravity
- \( Gd, Gu \) Centers of mass of the distal segment and of the upper arm, respectively
- \( i_{X_2}, i_{Y_2}, i_{Z_2} \) Unit vectors in the \( X_2, Y_2, \) and \( Z_2 \) directions, respectively
- \( i_{X_3}, i_{Y_3}, i_{Z_3} \) Unit vectors in the \( X_3, Y_3, \) and \( Z_3 \) directions, respectively
- \( I_{LU} \) Moment of inertia of the upper arm about its longitudinal axis
- \( I_{TD} \) Moment of inertia of the distal segment about any transverse axis through Gd
- \( I_{TU} \) Moment of inertia of the upper arm about any transverse axis through Gu
- \( l_U \) Vector pointing from the shoulder to the elbow
- \( m_D, m_U \) Masses of the distal segment and upper arm, respectively
- \( r_D \) Vector pointing from the elbow to Gd
- \( r_U \) Vector pointing from the shoulder to Gu
- \( R_1 \) Inertial reference frame attached to the ground
- \( R_2 \) Inertial reference frame whose origin coincides momentarily with Gd
- \( R_3 \) Inertial reference frame whose origin coincides momentarily with Gu
- \( T_D \) Torque exerted on the distal segment at the elbow
- \( T_U \) Torque exerted on the upper arm at the shoulder
- \( W_D, W_U \) Weights of the distal segment and upper arm, respectively
- \( X_{1,2,3}, Y_{1,2,3}, Z_{1,2,3} \) Vectors defining the orientations of the axes of reference frame \( R_1 \), \( R_2 \), and \( R_3 \)
- \( \alpha_{DX}, \alpha_{DY}, \alpha_{DZ} \) \( X_2, Y_2, \) and \( Z_2 \) components of \( \alpha_D \), respectively
- \( \alpha_{DGR} \) Motion-dependent component of \( \alpha_D \) associated with the force necessary to counter the acceleration of gravity
- \( \alpha_{DJT} \) Component of \( \alpha_D \) associated with \( T_D \)
- \( \alpha_{DSH} \) Motion-dependent component of \( \alpha_U \) associated with \( a_s \)
- \( \alpha_{DUA} \) Motion-dependent component of \( \alpha_U \) associated with \( \omega_U \)
- \( \alpha_{DUD} \) Motion-dependent component of \( \alpha_U \) associated with \( a_D \)
- \( \alpha_{UDT} \) Component of \( \alpha_U \) associated with the reaction to \( T_D \)
- \( \alpha_{UDV} \) Motion-dependent component of \( \alpha_U \) associated with \( \omega_D \)
- \( \alpha_{UGR} \) Motion-dependent component of \( \alpha_U \) associated with the force necessary to counter the acceleration of gravity
- \( \alpha_{UPJT} \) Component of \( \alpha_U \) associated with \( T_U \)
- \( \alpha_{USH} \) Motion-dependent component of \( \alpha_U \) associated with \( a_s \)
- \( \alpha_{UXi}, \alpha_{UYi}, \alpha_{UZi} \) \( X_3, Y_3, \) and \( Z_3 \) components of \( \alpha_U \), respectively
- \( \Sigma T_{Gd}, \Sigma T_{Gu} \) Sums of torques about Gd and Gu, respectively
- \( \omega_D, \omega_U \) Angular velocities of the distal segment and upper arm, respectively
- \( \omega_{UX}, \omega_{UY}, \omega_{UZ} \) \( X_3, Y_3, \) and \( Z_3 \) components of \( \omega_U \), respectively
is advisable to examine the motions of the adjacent segments in order to understand the origin of the joint forces and their effect on the motion of the segment being examined.

To investigate segmental interactions, some researchers have utilized simulation techniques to examine the motions of a kinetic chain when the value of a particular force or torque term that affects the motions of the segments in the chain is altered (Mena, Mansour, & Simon, 1981; Phillips, Roberts, & Huang, 1983). These methods allow the examination of the simulated motion and therefore provide valuable insight; however, used alone they do not provide direct information on the cause-effect mechanisms that influence the motions of the segments in the kinetic chain. Nevertheless, once the cause-effect mechanisms are understood, the simulation techniques will be useful for the evaluation of technique modifications.

Investigators have used a second type of methodology to study the energy transfers between the segments in a kinetic chain (Aleshinsky, 1986a, 1986b, 1986c, 1986d, 1986e; Joris, Edwards van Muyen, van Ingen Schenau, & Kemper, 1985; Robertson & Winter, 1980; Winter, 1979a, 1979b; Winter & Robertson, 1978). While these methods are valuable for the study of energy consumption and transfer, they do not permit the partitioning of the energy transfers between the segments into the various vector components of the alterations in the three-dimensional (3D) linear and angular velocities of the segments, since energy is a nonvector quantity. Therefore they seem to be of limited use for the investigation of the mechanisms involved in complex segmental interactions.

A third group of researchers have used models that express the joint forces of a kinetic chain as functions of the kinematic variables of segments in the chain (Chapman, Lonergan, & Caldwell, 1984; Hoy & Zernicke, 1985, 1986; Mena, Mansour, & Simon, 1981; Putnam, 1980, 1983; Wahrenburg, Lindbeck, & Ekholm, 1978; Winter & Robertson, 1978). These models allow the fractionation of the angular acceleration of each segment into terms associated with the muscular activities at its articulations, and terms associated with the motions of the segments of the kinetic chain. However, all of these models were derived for the analysis of two-dimensional (2D) motions, and in their present form they are unsuitable for the analysis of 3D motions.

The model presented in this paper fractionates the 3D angular acceleration vector of a segment into terms associated with the joint torques exerted on the segment, and into various terms associated with the kinematic variables of the segments in the kinetic chain ("motion-dependent" angular accelerations). The model was developed for a study of segmental interactions of the throwing arm during baseball pitching, and it is described in that context, but it can be used to analyze any two-segment 2D or 3D motion.

**Development of the Model**

*Description of the System*

The throwing arm system is shown in Figure 1a, and free-body diagrams of the distal segment (formed by the forearm, the hand, and the baseball) and of the upper arm are depicted in Figures 1b and 1c, respectively. Vector $l_v$ represents the position of the elbow relative to the shoulder; vector $r_v$ represents the position
Figure 1 — (a) Sketch of the throwing arm system; (b) free-body diagram of the distal segment; and (c) free-body diagram of the upper arm. S, E, and W represent the shoulder, elbow, and wrist, respectively.
of the center of mass (CM) of the upper arm (Gu) relative to the shoulder; and vector \( r_D \) represents the position of the CM of the distal segment (Gd) relative to the elbow.

The positions of the suprasternale, the right hip, shoulder, elbow, wrist, and third knuckle, the left hip and shoulder, and the center of the baseball, relative to inertial reference frame \( R_1 \) (Figure 2), were computed through 3D film analysis of several baseball pitches using the Direct Linear Transformation method of 3D cinematography (Abdel-Aziz & Karara, 1971; Walton, 1981). (For further explanations, see Feltner & Dapena, 1986.) The positions of Gu and Gd were computed from the landmark positions using Dempster (1955) cadaver data. The computed coordinates of the shoulder, elbow, and wrist landmarks were adjusted for each frame to force the magnitudes of \( l_u, r_u, \) and \( r_D \) to remain constant and equal to their respective average values throughout the pitch.

Dempster (1955) cadaver data were used to estimate the masses of the upper arm, forearm, and hand; the mass of the baseball was set at 0.17 kg. The mass of the distal segment was calculated as the sum of the masses of the forearm, hand, and baseball (prior to release), or of the forearm and hand only (after release). Whitsett (1963) moment of inertia data, personalized for each subject using a procedure described by Dapena (1978), were used to estimate the moments of inertia about the principal axes passing through the centers of mass of the upper arm, forearm, and hand. The moment of inertia of the baseball was set to zero. The moment of inertia of the distal segment about a transverse axis passing through its center of mass was computed using the Parallel Axis Theorem (Beer & Johnston, 1984b).

![Figure 2](image)

Figure 2 — Reference frame \( R_1 \) (Feltner & Dapena, 1986). The \( Z_1 \) axis was vertical; the \( X_1 \) axis was horizontal and pointed along the rear edge of the pitching rubber toward the third base side of the playing field; the \( Y_1 \) axis was perpendicular to \( X_1 \) and \( Z_1 \) and pointed toward home plate.
The distal segment was assumed to be subjected to the force of its own weight ($W_D$), and to a proximal force ($F_P$) and a couple ($T_P$) exerted at the elbow; the upper arm was assumed to be subjected to the force of its own weight ($W_U$), to a distal force ($-F_D$) and a couple ($-T_D$) exerted by the distal segment in reaction to $F_D$ and $T_D$, respectively, and to a force ($F_U$) and a couple ($T_U$) exerted at the shoulder. (The torques are indicated by double-headed vectors in Figures 1b and 1c.)

**Kinematic Data**

Quintic spline functions (developed by Wood & Jennings, 1979, and reported in detail by Vaughan, 1980) were used to smooth the time-dependent components of the 3D landmark coordinates and to compute the instantaneous velocity and acceleration values for each landmark (Feltner, 1984). These landmark kinematic data were then used to compute the inertial angular velocities and angular accelerations of the arm segments.

To compute the inertial angular velocity vector of each arm segment about a transverse axis ($\omega$), two vectors were defined at each instant: Vector $r$ pointed from the proximal to the distal endpoint of the segment, and $v_{rel}$ was the linear velocity of the distal endpoint relative to the proximal one. (Due to the constant lengths of the upper arm and distal segments, $v_{rel}$ was perpendicular to $r$.) The magnitude of $\omega$ was computed as the quotient of the magnitudes of $v_{rel}$ and $r$, and its direction was defined by the cross product of $r$ and $v_{rel}$. These angular velocities of the upper arm and of the distal segment were referred to as $\omega_U$ and $\omega_D$, respectively.

To calculate the inertial angular accelerations for the arm segments, the angular velocities about the longitudinal axis of each segment were assumed to be zero. It was not possible to compute this angular velocity component with sufficient accuracy for the distal segment, and in any case the close proximity between the baseball and the longitudinal axis of the segment indicated that this angular velocity component would have a negligible influence on the speed of the baseball. The angular velocity of the upper arm about its longitudinal axis could be computed with sufficient accuracy, but for the purposes of computing the angular acceleration of the upper arm it was decided to ignore it because it would have confounded the interpretation of the motion-dependent forces and angular accelerations (see the Appendix). The angular velocity of the upper arm about its longitudinal axis was still calculated, but it was used only to aid in the interpretation of the segment interaction data (see Feltner, 1989).

To compute the inertial angular accelerations of the upper arm and distal segments ($\alpha_U$ and $\alpha_D$, respectively), quintic spline functions were fitted, with zero smoothing, to the time-dependent values of each component of $\omega_U$ and $\omega_D$, respectively. The first derivatives of the functions provided the instantaneous values of $\alpha_U$ and $\alpha_D$, respectively.

**Motion-Dependent Forces**

According to the principles of relative motion (Beer & Johnston, 1984a), the acceleration of the CM of the upper arm ($a_{cm}$) was expressed as

$$a_{cm} = a_s + (\alpha_U \times r_U) + (\omega_U \times (\omega_U \times r_U))$$  (1)
where $a_s$ was the linear acceleration of the right (throwing) shoulder. The acceleration of the CM of the distal segment ($a_{gd}$) was computed as

$$a_{gd} = a_s + (\alpha_u \times l_u) + (\omega_u \times (\omega_u \times l_u)) + (\alpha_d \times r_d) + (\omega_d \times (\omega_d \times r_d))$$

(2)

The following equation was written for the sum of the forces exerted on the distal segment:

$$m_D a_{gd} = F_D + m_D g$$

(3)

where $m_D$ is the mass of the distal segment and $g$ is the acceleration of gravity. Substituting Equation 2 for $a_{gd}$ in Equation 3 and rearranging the terms resulted in the following expression for the force exerted on the distal segment at the elbow ($F_D$):

$$F_D = m_D a_s + m_D (\omega_u \times (\omega_u \times l_u)) + m_D (\omega_u \times (\omega_u \times r_d)) + m_D (\alpha_u \times l_u) + m_D (\alpha_d \times r_d) - m_D g$$

(4)

The following equation was written for the sum of the forces exerted on the upper arm:

$$m_U a_{gu} = F_U - F_D + m_U g$$

(5)

where $m_U$ is the mass of the upper arm. Equation 1 was substituted for $a_{gu}$ and Equation 4 for $F_D$ in Equation 5. Rearrangement of the terms in the resulting equation produced the following expression for the force exerted on the upper arm at the shoulder ($F_U$):

$$F_U = m_D a_s + m_U a_s + m_D (\omega_u \times (\omega_u \times l_u)) + m_U (\omega_u \times (\omega_u \times r_d)) + m_D (\alpha_u \times l_u) + m_U (\alpha_u \times r_u) + m_D (\alpha_d \times r_d) - m_D g - m_U g$$

(6)

Equations 4 and 6 expressed $F_D$ and $F_U$, respectively, as sums of six motion-dependent forces. The terms labeled (a) through (f) on the right side of the equations were motion-dependent forces associated with (a) linear acceleration of the right shoulder, (b) the angular velocity of the upper arm, (c) the angular velocity of the distal segment, (d) the angular acceleration of the upper arm, (e) the angular acceleration of the distal segment, and (f) the force necessary to counter the acceleration of gravity.

**Sum of Torques**

For the computation of the sum of the torques acting on the distal and upper arm segments, and later to provide anatomically relevant meaning to the computed torques and angular accelerations, two new reference frames were defined for
each instant of the pitch in terms of reference frame \( R_1 \). Reference frame \( R_2 \) was static, but its origin coincided momentarily with the CM of the distal segment and its axes with the principal axes of the distal segment at the time of each frame (Figure 3a). The axes of \( R_2 \) were defined by three vectors: Vector \( Y_2 \) pointed from the wrist to the elbow; \( X_2 \) was the cross product of \( I_U \) and \( Y_2 \); \( Z_2 \) was the cross product of \( X_2 \) and \( Y_2 \) (Figure 3a).

Reference frame \( R_3 \) also was static. Its origin coincided with the CM of the upper arm and its axes with the principal axes of the upper arm at the time of each frame (Figure 3b). The axes were defined by three vectors: Vector \( X_3 \) pointed from the elbow to the shoulder; \( Y_3 \) was the cross product of \( X_3 \) and a vector pointing from the suprasternale to the mid-hip point; \( Z_3 \) was the cross product of \( X_3 \) and \( Y_3 \) (Figure 3b).

Euler's Equation of Motion (Ginsberg & Genin, 1984) was used to compute the sum of the torques acting about the CM of each segment. For the distal segment, the components of \( \omega_D \) and \( \alpha_D \) in the \( Y_2 \) direction were zero, and the principal moments of inertia in the \( X_2 \) and \( Z_2 \) directions were identical and equal to \( I_{T_D} \) (the moment of inertia about any transverse axis of the distal segment). Therefore, at each instant of the pitch the sum of the torques about the CM of the distal segment \( \Sigma T_{G_d} \) could be computed as

\[
\Sigma T_{G_d} = [I_{T_D}\alpha_{Dx}] i_2 + \text{zero} j_2 + [I_{T_D}\alpha_{Dz}] k_2
\]  

where \( i_2 \), \( j_2 \), and \( k_2 \) are unit vectors in the \( X_2 \), \( Y_2 \), and \( Z_2 \) directions, respectively, and \( \alpha_{Dx} \) and \( \alpha_{Dz} \) are the components of \( \alpha_D \) in the \( X_2 \) and \( Z_2 \) directions, respectively.

For the upper arm, the components of \( \omega_U \) and \( \alpha_U \) in the \( X_3 \) direction were zero, and the principal moments of inertia in the \( Y_3 \) and \( Z_3 \) directions were identical and equal to \( I_{T_U} \) (the moment of inertia about any transverse axis of the upper arm). This resulted in the following expression for \( \Sigma T_{G_u} \):

\[
\Sigma T_{G_u} = \text{zero} i_3 + [I_{T_U}\alpha_{UY}] j_3 + [I_{T_U}\alpha_{UZ}] k_3
\]

where \( i_3 \), \( j_3 \), and \( k_3 \) are unit vectors in the \( X_3 \), \( Y_3 \), and \( Z_3 \) directions, respectively, and \( \alpha_{UY} \) and \( \alpha_{UZ} \) are the components of \( \alpha_U \) in the \( Y_3 \) and \( Z_3 \) directions, respectively.

**Fractionated Segment Torques**

The proximal joint torques made on the distal segment \( (T_D) \) and on the upper arm segment \( (T_U) \) were computed using the following equations:

\[
T_D = \Sigma T_{Gd} + (r_D \times F_D)
\]

\[
T_U = \Sigma T_{Gu} + T_D + (r_U \times F_U) + ((r_U - r_D) \times F_D)
\]

These computations, as well as those leading ultimately to the fractionated angular accelerations of the segments, were performed with parameters expressed in terms of reference frame \( R_1 \). However, in order to provide anatomical relevance to the data generated by the model, all data for the distal segment and the upper arm were finally expressed in terms of reference frames \( R_2 \) and \( R_3 \), respectively, for the instant of each frame (Feltner & Dapena, 1986). To assist the reader, the remaining equations for the torques and angular accelerations also will be expressed in terms of these two reference frames for the distal segment and upper arm, respectively.
Figure 3 — (a) Reference frame $R_2$: Torque and angular acceleration terms in the $X_2$, $Y_2$, and $Z_2$ directions were associated with the anatomical directions of flexion/extension, pronation/supination, and valgus/varus rotation at the elbow, respectively.

(b) Reference frame $R_3$: Torque and angular acceleration terms in the $X_3$, $Y_3$, and $Z_3$ directions were associated with the anatomical directions of internal/external rotation, abduction/adduction, and horizontal abduction/adduction at the shoulder, respectively.
Equation 7 was substituted for $\Sigma T_{Gd}$ and Equation 4 for the $F_D$ in Equation 9 to produce an equation that expressed the resultant torque exerted about the CM of the distal segment as a function of various terms. The term containing $\omega_D$ created no torque about $G_d$, and therefore it was zero. There were no torque components in the $Y_2$ direction, and the term containing $\alpha_D$ reduced to two terms, $m_d (r_D)^2 \alpha_{DX}$ and $m_d (r_D)^2 \alpha_{DZ}$, in the $X_2$ and $Z_2$ directions, respectively. The resulting equation then was transformed into two scalar equations that expressed the torques exerted about $G_d$ in the $X_2$ (flexion/extension) and $Z_2$ (valgus/varus rotation) directions, respectively:

\[ [I_{TD} + m_d (r_D)^2] \alpha_{DX} = T_D \cdot i_2 - (r_d \times m_d a_d) \cdot i_2 - (r_d \times m_d (\alpha_U \times l_U)) \cdot i_2 - (r_d \times m_d (\omega_U \times (\omega_U \times l_U))) \cdot i_2 + (r_d \times m_d g) \cdot i_2 \]  

(11)

\[ [I_{TD} + m_d (r_D)^2] \alpha_{DZ} = T_D \cdot k_2 - (r_d \times m_d a_d) \cdot k_2 - (r_d \times m_d (\alpha_U \times l_U)) \cdot k_2 - (r_d \times m_d (\omega_U \times (\omega_U \times l_U))) \cdot k_2 + (r_d \times m_d g) \cdot k_2 \]  

(12)

(The dots in Equations 11–20 indicate dot products.)

Equation 8 was substituted for $\Sigma T_{Gu}$, Equation 6 for $F_U$, and Equation 4 for $F_D$ in Equation 10. The terms in the resulting vectorial equation then were rearranged and simplified (see Feltner, 1987, for details). This resulted in three scalar equations, for the $X_3$ (internal/external rotation), $Y_3$ (abduction/adduction), and $Z_3$ (horizontal abduction/adduction) directions, respectively:

\[ I_{LU} \alpha_{UX} = \text{zero} = T_U \cdot i_3 - T_D \cdot i_3 \]  

(13)

\[ [I_{TU} + m_u (r_u)^2 + m_d (l_u)^2] \alpha_{UY} = T_U \cdot j_3 - T_D \cdot j_3 - (l_U \times m_d a_s + r_u \times m_u a_d) \cdot j_3 - (l_U \times m_d (\alpha_D \times r_D)) \cdot j_3 - (l_U \times m_d (\omega_D \times (\omega_D \times r_D))) \cdot j_3 + (l_U \times m_d g + r_u \times m_u g) \cdot j_3 \]  

(14)

\[ [I_{TU} + m_u (r_u)^2 + m_d (l_u)^2] \alpha_{UZ} = T_U \cdot k_3 - T_D \cdot k_3 - (l_U \times m_d a_s + r_u \times m_u a_d) \cdot k_3 - (l_U \times m_d (\alpha_D \times r_D)) \cdot k_3 - (l_U \times m_d (\omega_D \times (\omega_D \times r_D))) \cdot k_3 + (l_U \times m_d g + r_u \times m_u g) \cdot k_3 \]  

(15)

where $I_{LU}$ is the moment of inertia about the longitudinal axis of the upper arm.

Equation 13 expressed the torques exerted about $G_u$ in the $X_3$ (internal/external rotation) direction. The left side of this equation was zero because $\alpha_{UX}$, the component of $\alpha_u$ in the $X_3$ direction, was implicitly set to zero when the angular velocity about the longitudinal axis of the upper arm was fixed to zero for the purposes of computing $\alpha_U$. The rationale for this assumption is presented in the Appendix.
Equations 14 and 15 expressed the torques exerted about Gu in the Y₃ (abduction/adduction) and Z₃ (horizontal abduction/adduction) directions, respectively.

**Fractionated Angular Accelerations**

The torque terms in Equations 11 through 15 were divided by the inertial parameters on the left side of each equation to express the angular accelerations of the upper arm or distal segment that would be associated with each term. For the distal segment, this resulted in two scalar equations for the X₂ and Z₂ directions:

\[
\alpha_{DX} = \frac{(T_D)/[I_{TD} + m_D (r_D)^2]}{[I_{TD} + m_D (r_D)^2]} \cdot i_2 \tag{16}
\]

\[
- (r_D \times m_D a_\theta) / [I_{TD} + m_D (r_D)^2] \cdot i_2 \tag{a}
\]

\[
- (r_D \times m_D (\alpha_u \times l_U)) / [I_{TD} + m_D (r_D)^2] \cdot i_2 \tag{b}
\]

\[
- (r_D \times m_D (\omega_U \times (\omega_U \times l_U))) / [I_{TD} + m_D (r_D)^2] \cdot i_2 \tag{c}
\]

\[
+ (r_D \times m_D g) / [I_{TD} + m_D (r_D)^2] \cdot i_2 \tag{d}
\]

\[
\alpha_{DZ} = \frac{(T_D)/[I_{TD} + m_D (r_D)^2]}{[I_{TD} + m_D (r_D)^2]} \cdot k_2 \tag{17}
\]

\[
- (r_D \times m_D a_\theta) / [I_{TD} + m_D (r_D)^2] \cdot k_2 \tag{a}
\]

\[
- (r_D \times m_D (\alpha_u \times l_U)) / [I_{TD} + m_D (r_D)^2] \cdot k_2 \tag{b}
\]

\[
- (r_D \times m_D (\omega_U \times (\omega_U \times l_U))) / [I_{TD} + m_D (r_D)^2] \cdot k_2 \tag{c}
\]

\[
+ (r_D \times m_D g) / [I_{TD} + m_D (r_D)^2] \cdot k_2 \tag{d}
\]

The five angular acceleration terms at the right of the equations, labeled (a) through (e), represented the component of each term of the angular acceleration of the distal segment in the X₂ and Z₂ directions, respectively. The terms were as follows: (a) $\alpha_{DPT}$, the angular acceleration associated with the proximal joint torque exerted on the distal segment; (b) $\alpha_{PSH}$, the motion-dependent angular acceleration associated with the linear acceleration of the right shoulder; (c) $\alpha_{DUA}$, the motion-dependent angular acceleration associated with the angular acceleration of the upper arm; (d) $\alpha_{DUV}$, the motion-dependent angular acceleration associated with the angular velocity of the upper arm; (e) $\alpha_{DGR}$, the motion-dependent angular acceleration associated with the force necessary to counter the acceleration of gravity.

The upper arm required three scalar equations to express the components of $\alpha_u$ in the X₃, Y₃, and Z₃ directions:

\[
\alpha_{UX} = \text{zero} = \frac{T_U/I_{LU}}{I_{LU}} \cdot i_3 \tag{18}
\]

\[
- T_D/I_{LU} \cdot i_3 \tag{a}
\]

\[
\alpha_{UY} = \frac{T_U/I_{TU} + m_U (r_U)^2 + m_D (l_U)^2}{I_{TU} + m_U (r_U)^2 + m_D (l_U)^2} \cdot j_3 \tag{19}
\]

\[
- (l_U \times m_D a_\theta + r_U \times m_U a_\theta) / [I_{TU} + m_U (r_U)^2 + m_D (l_U)^2] \cdot j_3 \tag{a}
\]

\[
- (l_U \times m_D (\alpha_D \times r_D)) / [I_{TU} + m_U (r_U)^2 + m_D (l_U)^2] \cdot j_3 \tag{b}
\]

\[
- (l_U \times m_D (\omega_D \times (\omega_D \times r_D))) / [I_{TU} + m_U (r_U)^2 + m_D (l_U)^2] \cdot j_3 \tag{c}
\]

\[
+ (l_U \times m_D g + r_U \times m_U g) / [I_{TU} + m_U (r_U)^2 + m_D (l_U)^2] \cdot j_3 \tag{d}
\]

\[
\alpha_{UZ} = \frac{T_D/I_{TU} + m_U (r_U)^2 + m_D (l_U)^2}{I_{TU} + m_U (r_U)^2 + m_D (l_U)^2} \cdot k_3 \tag{e}
\]

\[
- (l_U \times m_D a_\theta + r_U \times m_U a_\theta) / [I_{TU} + m_U (r_U)^2 + m_D (l_U)^2] \cdot k_3 \tag{f}
\]

\[
- (l_U \times m_D (\omega_D \times (\omega_D \times r_D))) / [I_{TU} + m_U (r_U)^2 + m_D (l_U)^2] \cdot k_3 \tag{g}
\]

\[
+ (l_U \times m_D g + r_U \times m_U g) / [I_{TU} + m_U (r_U)^2 + m_D (l_U)^2] \cdot k_3 \tag{h}
\]
The two angular acceleration terms on the right side of Equation 18, labeled (a) and (b), represented (a) \( \alpha_{UPT} \), the X₃ component of \( \alpha_U \) associated with the proximal joint torque exerted on the upper arm; (b) \( \alpha_{UDT} \), the X₃ component of \( \alpha_U \) associated with the reaction to the proximal joint torque made on the distal segment.

The six angular acceleration terms on the right side of Equations 19 and 20, labeled (a) through (f), represented only the components of each term in the Y₃ and Z₃ directions, respectively: (a) \( \alpha_{UPT} \), the angular acceleration associated with the proximal joint torque exerted on the upper arm; (b) \( \alpha_{UDT} \), the angular acceleration associated with the reaction to the proximal joint torque exerted on the distal segment; (c) \( \alpha_{USH} \), the motion-dependent angular acceleration associated with the linear acceleration of the right shoulder; (d) \( \alpha_{UDA} \), the motion-dependent angular acceleration associated with the angular acceleration of the distal segment; (e) \( \alpha_{UDV} \), the motion-dependent angular acceleration associated with the angular velocity of the distal segment; (f) \( \alpha_{UGR} \), the motion-dependent angular acceleration associated with the force necessary to counter the acceleration of gravity.

Discussion

In previous research, a number of investigators have examined the interactions between segments in a kinetic chain by fractionating the angular acceleration of each segment into terms associated with the muscular activities at its articulations and into terms associated with the kinematics of the segments in the kinetic chain (Chapman et al., 1984; Hoy & Zernicke, 1985, 1986; Mena et al., 1981; Putnam, 1980, 1983; Wahrenburg et al., 1978; Winter & Robertson, 1978). However, all of the models used in these investigations were restricted to the analysis of 2D motions. The model developed in the present paper has a distinct advantage in that it can be used to analyze any 2D or 3D motion.

In the present model, the values of the angular acceleration terms in each of the anatomically relevant directions for the upper arm and distal segment indicate the contribution of each term to the net angular acceleration of the segment, and consequently its effect on the angular velocity of the segment. Thus, by examining the angular acceleration data together with selected kinematic data defining the orientations and angular velocities of the segments in the kinetic chain, the motions of the segments can be linked with the causal factors that are responsible for producing these motions. Specifically, it can be determined whether the angular acceleration of a segment is due to (a) the joint torques exerted at its articulations, (b) the linear acceleration of the proximal endpoint in the kinetic chain, (c) gravity, or (d) the angular velocity or angular acceleration of the adjacent segment in the kinetic chain. An application of this model to the 3D motions of the throwing arm during baseball pitching is presented in the companion paper (see Feltner, 1989).
References


To compute the inertial angular acceleration of the upper arm, it was assumed that the angular velocity of the upper arm about its longitudinal axis was zero. The advantages of this assumption can best be illustrated with an example.

Consider a ball moving in a circular path with constant angular velocity ($\omega$) in a horizontal plane defined by axes $X_A$ and $Z_A$ of reference frame $R_A$ (Figures A1a and b). At the instant depicted, the linear acceleration of the center of the ball ($a_B$) is

$$a_B = |\omega \times (\omega \times r)| i_A + \text{zero } j_A + \text{zero } k_A$$

where $i_A$, $j_A$, and $k_A$ are unit vectors in the respective directions of the $X_A$, $Y_A$, and $Z_A$ axes of reference frame $R_A$.

Next, let us attach one end of a slender rod to the ball, and the other end to a point
Figure A1 — (a) Overhead view (\(-Z_A \text{ vs. } X_A\)) and (b) side view (\(Y_A \text{ vs. } X_A\)) sketches depicting the motion of the center of the ball (B) and its linear acceleration, \(a_B\) (see text).

P located directly below C (in the negative \(Y_A\) direction in Figure A2), while the center of the ball keeps moving in the same way as in Figure A1. Vector \(R\) indicates the orientation of the longitudinal axis of the rod.

Let us now assume that the rod cannot rotate about the axis defined by \(R\). As the ball follows its circular path about C, the rod will have an angular velocity (\(\omega_I\) in Figure A2) associated with its conical motion about a line joining P and C, and \(\omega_I\) will always be perpendicular to \(R\) and contained in the plane defined by \(R\) and the line joining P and C. Since the direction of \(\omega_I\) will be changing constantly, the rod also will have an angular acceleration (\(\alpha_I\)), and this angular acceleration will be directed in the negative \(Z_A\) direction at the instant shown in Figure A2. The linear acceleration of B can now be expressed as

\[
a_B = (\omega_I \times (\omega_I \times R)) + (\alpha_I \times R)
\]

(A2)

where the terms \(\omega_I \times (\omega_I \times R)\) and \(\alpha_I \times R\) are the centripetal and tangential accelerations of B relative to P, and they are shown by vectors \(a_{IC}\) and \(a_{IT}\), respectively, in Figure A2. Since \(a_B\) must have the same value as in the situation depicted in Figure A1, the \(j_A\) components of \(a_{IC}\) and \(a_{IT}\) have to be equal and opposite to each other, and the \(i_A\) components of \(a_{IC}\) and \(a_{IT}\) must add up to \(|a_B| \ i_A\).
Let us now consider a third situation, where the motion of the center of the ball remains the same as in the previous two situations, but the form of attachment of the rod at point P is changed slightly: The longitudinal axis of the rod continues to rotate in a conical motion with angular velocity $\omega_l$, but now the rod also rotates about its longitudinal axis (R) with an angular velocity of constant magnitude ($\omega_L$ in Figure A3).

As $\omega_L$ will not affect the motion of the center of the ball, it will not affect $a_B$. However, the net angular velocity of the rod ($\omega_2$ in Figure A3) is no longer perpendicular to R, and the angular acceleration of the rod now has an additional component ($\alpha_L$ in Figure A3) that is associated with the change in the direction of $\omega_L$. The net angular acceleration of the rod ($\alpha_2$ in Figure A3) is the sum of $\alpha_l$ and $\alpha_L$. The linear acceleration of the center of the ball can now be expressed as

$$a_B = (\omega_2 \times (\omega_2 \times R)) + (\alpha_2 \times R)$$  \hspace{1cm} (A3)

$$a_B = (\omega_2 \times (\omega_2 \times R)) + (\alpha_l \times R) + (\alpha_L \times R)$$  \hspace{1cm} (A4)

The term $\omega_2 \times (\omega_2 \times R)$ ($a_{2C}$ in Figure A3) is no longer aligned with R, and thus the centripetal acceleration of B relative to P makes a different contribution to the acceleration of the rod than when $\omega_L$ was zero. Also, the term $\alpha_2 \times R$ ($a_{2T}$ in Figure A3) will be different from the term ($\alpha_l \times R$), due to the effect of the added term $\alpha_L \times R$. However, as Equations A3 and A4 must still correctly express $a_B$, the sum of $a_{2C}$ and $a_{2T}$ must be the same as the sum of $a_{1C}$ and $a_{1T}$, and therefore the amount that $a_{2C}$ deviates from $a_{1C}$ (vector $da$ in Figure A3) must be compensated for by the term $\alpha_L \times R$. 

Figure A2 — Sketch showing the vectors used to compute $a_{1C}$ and $a_{1T}$ (see text).
In sum, by including $\omega_L$ in the equations used to compute the linear acceleration of B ($a_B$), an extra term ($da$) is introduced, with opposite signs, into the centripetal and tangential components of $a_B$. The addition of this extra term prevents the "centripetal" component from pointing in the radial direction from B to P. Therefore, the inclusion of $\omega_L$ does not affect the motion of the center of the ball, but it leads to centripetal and tangential acceleration values that differ from the generally accepted concepts of centripetal and tangential. Consequently, to maintain the generally accepted meaning of the centripetal and tangential components of acceleration, it is necessary to ignore $\omega_L$ (and consequently, $\alpha_L$, the angular acceleration term associated with $\omega_L$). Thus, to compute the angular acceleration of the upper arm, the angular velocity about the longitudinal axis of the upper arm was implicitly set to zero.

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